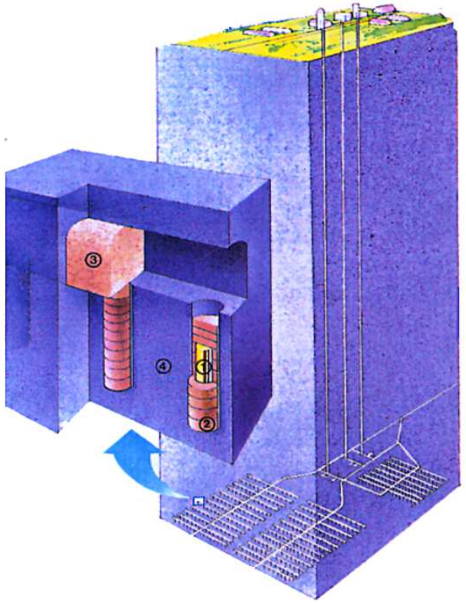
Damage and Fracture: Numerical Approaches at the Structure Scale

Eric Lorentz (EDF R&D)

SOME TARGET STRUCTURES AT EDF



OBJECTIVES

Characteristics

- Large structures
- Large crack propagation
- Various materials

Restricted framework

- Quasi-static tensile loading
- Scale of a single macroscopic crack

Quantities of interest

- Maximal load, potential instabilities
- Crack path, length and opening (tightness)

(Tate Modern, Londres)

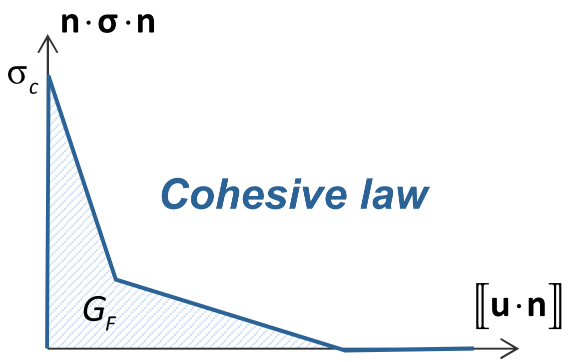
Computation

- Robustness
- Reliability
- Performance

AVAILABLE MODELING TOOLS

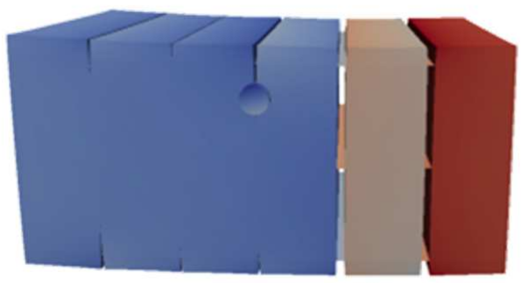
Cohesive zone model (CZM)

Fictitious crack model



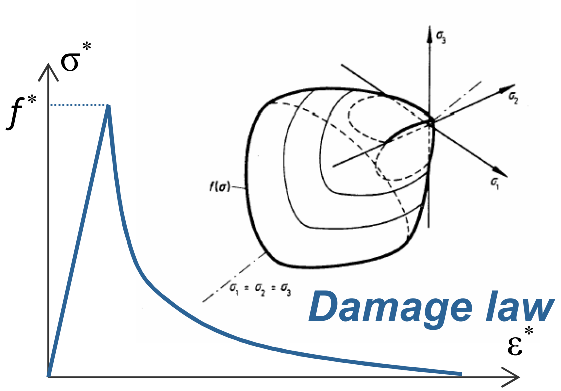
Engineer parameters (σ_c , G_F)

Crack modeling: opening, length, contact



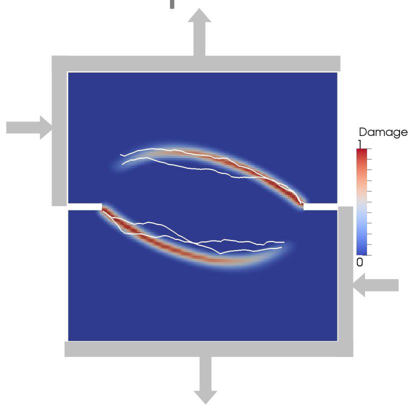
Continuum Damage Mechanics (CDM)

Smeared crack model



Damage threshold in the stress space

Crack path description



OUTLINE

Cohesive Zone Models (CZM)

- Principle and basic finite elements
- Extrinsic and intrinsic laws – Numerical consequences
- Interface mixed finite elements
- Instabilities and path-following methods
- Crack path prediction and related issues

Continuum Damage Mechanics (CDM)

- Strain-softening, localisation and constitutive nonlocality
- Nonlocal constitutive relations
- Gradient models: formulation and mixed finite elements
- Anisotropy vs isotropy and damage – stiffness coupling
- Vanishing internal length and the cohesive limit

MY PHILOSOPHY REGARDING NUMERICAL STRATEGIES

1. Avoid damage computations if post-treatment criteria are applicable

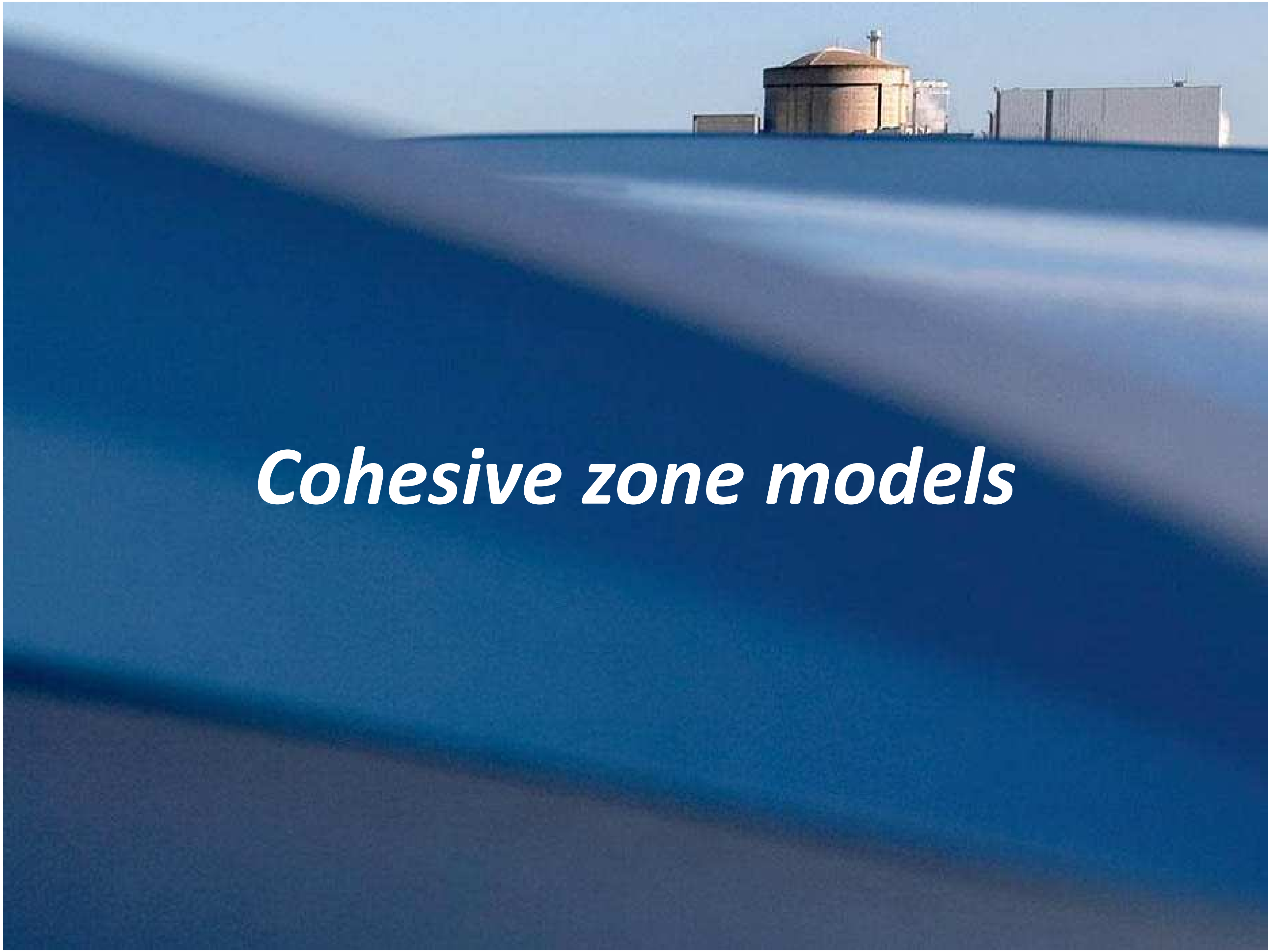
- Energy release rate G, path integral J
- Rice and Tracey growth criterion

2. If you can guess potential crack paths, use Cohesive Zone Models

- No stiffness regularisation (extrinsic laws)
- Mixed finite elements along the crack path
- Path-following methods if instabilities are expected

3. Otherwise rely on Continuum Damage Mechanics

- Nonlocal constitutive laws (preferably gradient models)
- Refined mesh in the damaged areas
- High computational cost


Cohesive zone models

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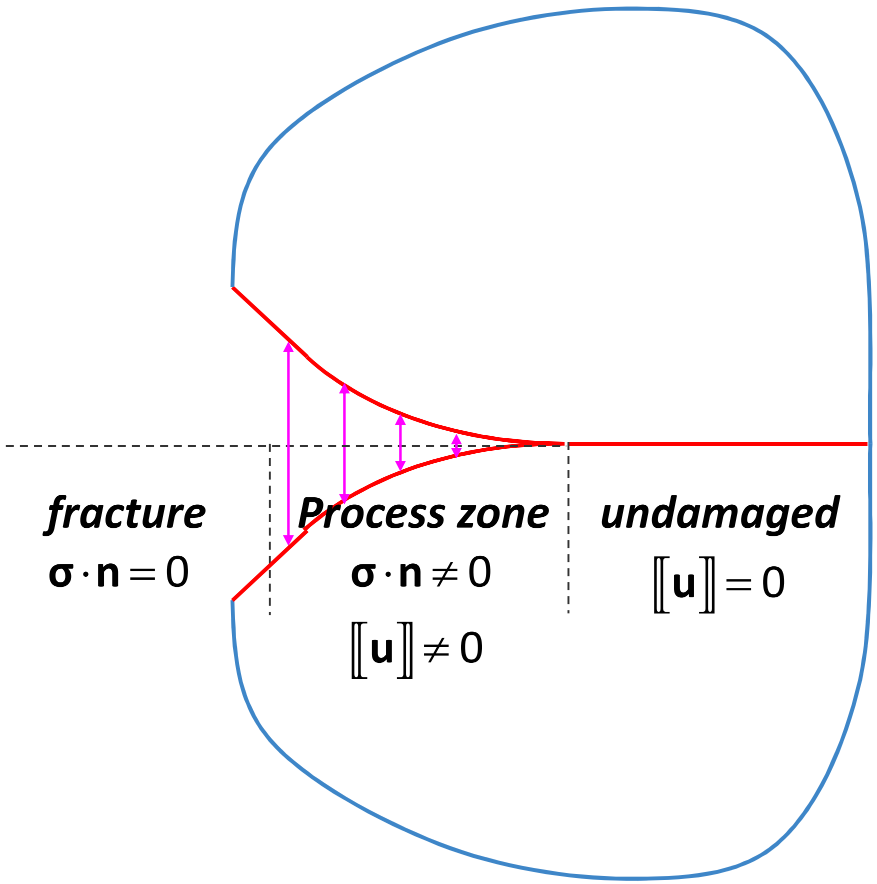
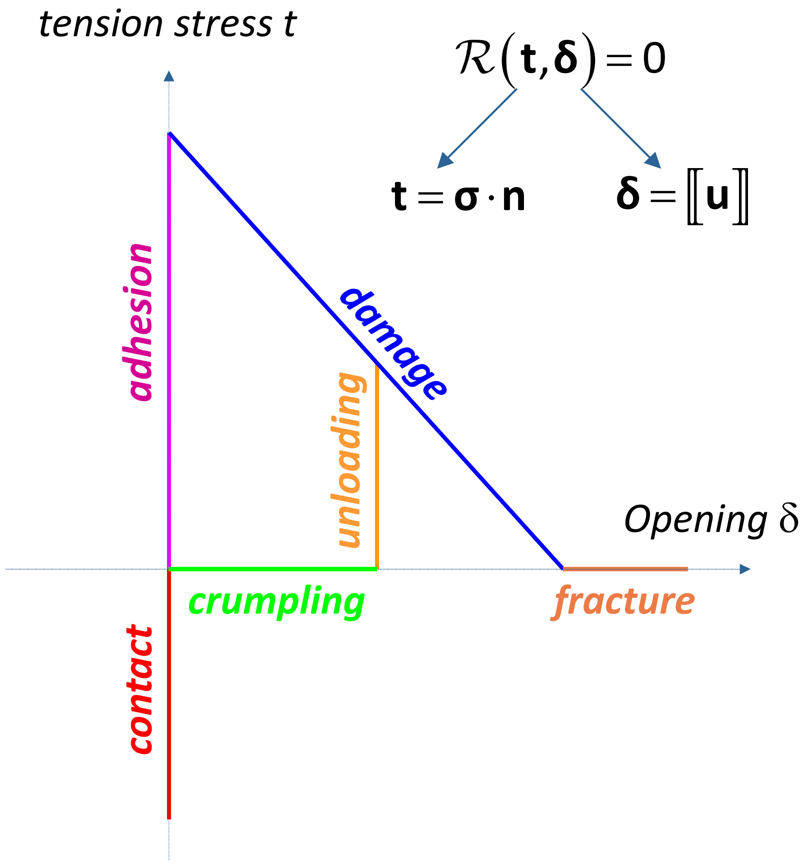
COHESIVE FRACTURE: PRINCIPLE

Process zone



Cohesive law



ADHESION AND STRESS THRESHOLD

Initial adhesion

$$\forall\, \mathbf{x} \in \Omega\,;\, \forall\, \mathbf{n} \quad (\sigma_n, \boldsymbol{\sigma}_t) \in \mathcal{D}_A$$

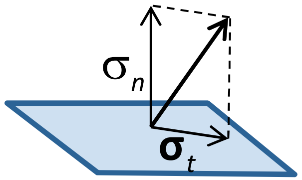
$$\mathcal{D}_A = \left\{ \, (\sigma_n, \boldsymbol{\sigma}_t) \;;\; \sigma_n \leq f_t \, \right\}$$

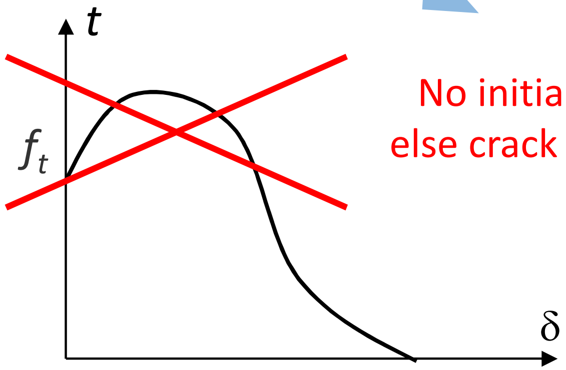
Criterion on the stress tensor

$$\forall\, \mathbf{x} \in \Omega \quad \mathrm{eig}\big(\boldsymbol{\sigma}(\mathbf{x})\big) \in \hat{\mathcal{D}}_A$$

$$\hat{\mathcal{D}}_A = \left\{ \, \boldsymbol{\sigma} \;;\; \max \sigma_i \leq f_t \, \right\}$$

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \sigma_n \mathbf{n} + \boldsymbol{\sigma}_t$$





No initial increase
else crack continuum

ENERGY FORMULATION AND BASIC FINITE ELEMENT

Energy minimisation

Pedagogic case

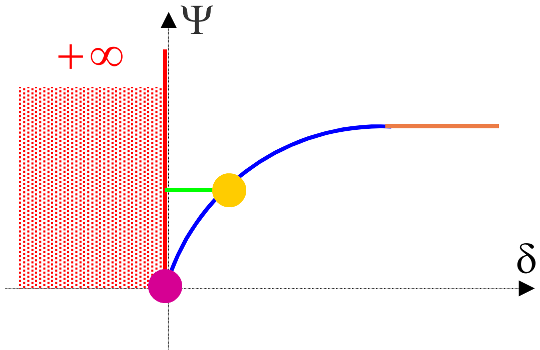
Total energy

$$\mathcal{E}(\mathbf{u},\boldsymbol{\delta}) \equiv \mathcal{E}_{\text{fr}}(\boldsymbol{\delta}) + \mathcal{E}_{\text{el}}(\mathbf{u}) - \mathcal{W}_{\text{ext}}(\mathbf{u})$$

$$\min_{\boldsymbol{\delta}=[\![\mathbf{u}]\!]} \mathcal{E}(\mathbf{u},\boldsymbol{\delta})$$

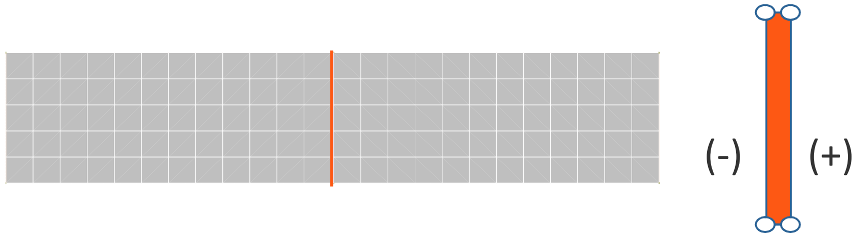
Fracture energy and cohesive law

$$\mathcal{E}_{fr}(\boldsymbol{\delta}) \equiv \int_{\Gamma} \Psi(\boldsymbol{\delta}(\mathbf{x}))\,ds \quad \rightarrow \quad \mathbf{t} = \frac{\partial \Psi}{\partial \boldsymbol{\delta}}$$



Basic spatial discretisation

Cohesive crack between bulk elements



$$\mathbf{u}(\mathbf{x}) = \sum_n \mathrm{N}^n(\mathbf{x})\mathbf{U}_n$$

$$\boldsymbol{\delta}(\mathbf{s}) = [\![\mathbf{u}]\!](\mathbf{s}) = \sum_n \left[\mathrm{N}^n(\mathbf{s}^+) - \mathrm{N}^n(\mathbf{s}^-)\right]\mathbf{U}_n$$

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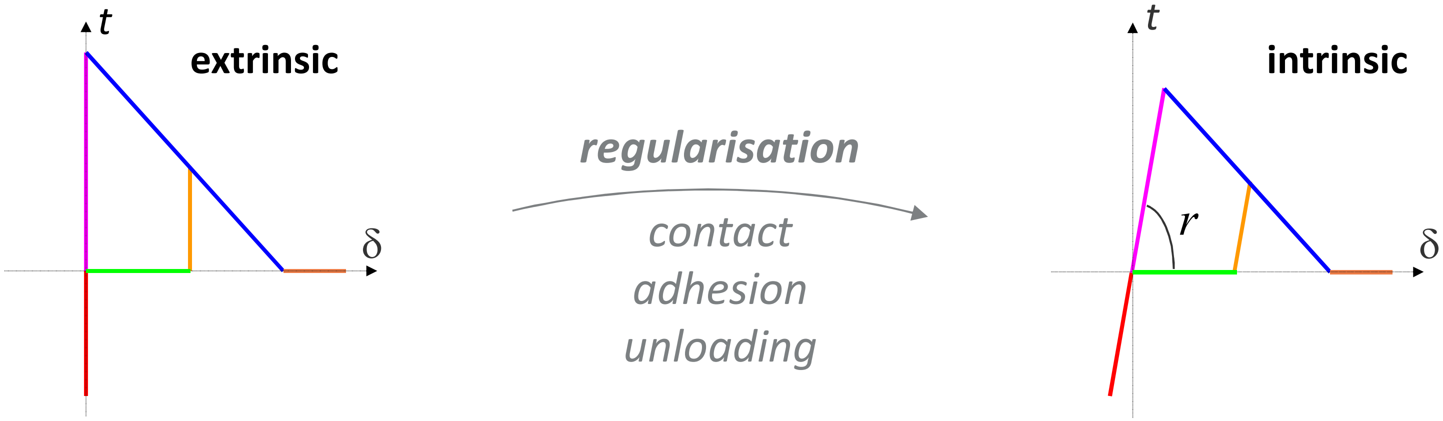
Extrinsic and intrinsic cohesive laws



$t(\delta)$??? $t(\delta)$ OK

Energy interpretation (contact case)

penalty

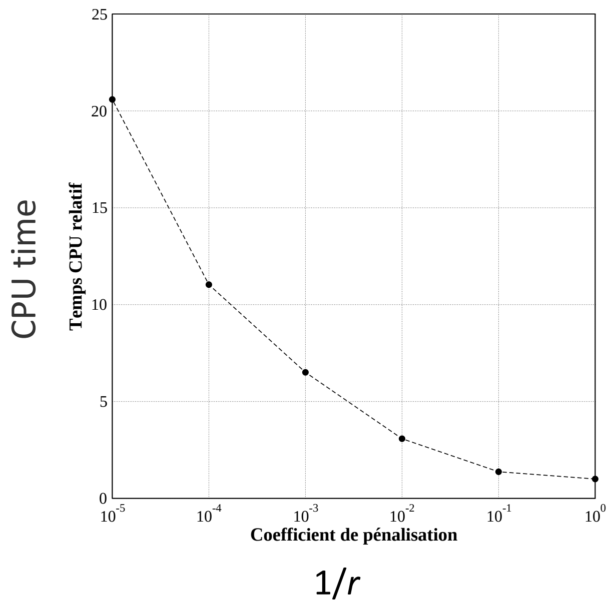
$$\min_{\delta \geq 0} \int_\Gamma \Psi(\delta)\,ds \;+\; \ldots \quad \longrightarrow \quad \min_{\delta} \int_\Gamma \frac{r}{2}\langle\delta\rangle_-^2 + \Psi(\delta)\,ds \;+\; \ldots$$

PENALTY-INDUCED NUMERICAL DIFFICULTIES

Bad numerical conditioning

Choice of the penalty parameter *r*
A trade-off between:

1. Accuracy (?)
2. Performance (and even robustness)



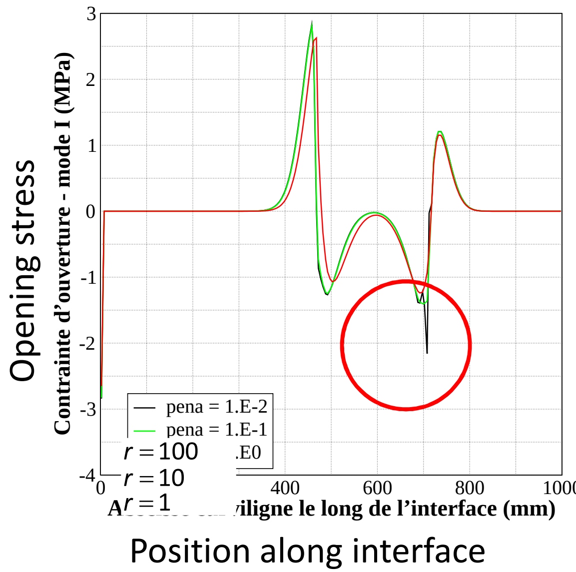
Bending of sandwich beams

Ill-posed asymptotic problem

The asymptotic problem for $r \to \infty$ is ill-posed (LBB condition not fulfiled)

$\Downarrow$

Spurious stress oscillations



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LAGRANGIAN RELAXATION

Objective

$$\min_{a}\left[\mathrm{R}(a)+\mathrm{S}(a)\right]$$ *where S is a "troublesome function"*

Steps

1. *Decomposition* $\min_{a=b} \mathrm{R}(a)+\mathrm{S}(b)$
2. *Augmentation* $\min_{a=b}\left[\mathrm{R}(a)+\mathrm{S}(b)+\frac{r}{2}(a-b)^2\right]$
3. *Dualisation* $\min_{a}\max_{\lambda}\min_{b}\left[R(a)+S(b)+\frac{r}{2}(a-b)^2+\lambda(a-b)\right]$
4. *Discretisation* *collocation for b $\rightarrow$ pointwise elimination*

Resulting in a mixed finite element (dof *a* and λ)

A MIXED FINITE ELEMENT FOR EXTRINSIC COHESIVE LAWS

Lagrangian relaxation

$$\mathcal{E}(\mathbf{u},\boldsymbol{\delta}) \equiv \mathcal{E}_{\text{fr}}(\boldsymbol{\delta}) + \mathcal{E}_{\text{el}}(\mathbf{u}) - \mathcal{W}_{\text{ext}}(\mathbf{u})$$

$$\min_{\boldsymbol{\delta}=[\![\mathbf{u}]\!]} \mathcal{E}(\mathbf{u},\boldsymbol{\delta})$$

$$\mathcal{L}(\mathbf{u},\boldsymbol{\delta},\boldsymbol{\lambda}) \equiv \mathcal{E}(\mathbf{u},\boldsymbol{\delta}) + \int_\Gamma \boldsymbol{\lambda}\cdot\left([\![\mathbf{u}]\!]-\boldsymbol{\delta}\right) + \frac{r}{2}\int_\Gamma \left([\![\mathbf{u}]\!]-\delta\right)^2$$

$$\min_{\mathbf{u}} \max_{\boldsymbol{\lambda}} \min_{\boldsymbol{\delta}} \mathcal{L}(\mathbf{u},\boldsymbol{\delta},\boldsymbol{\lambda})$$

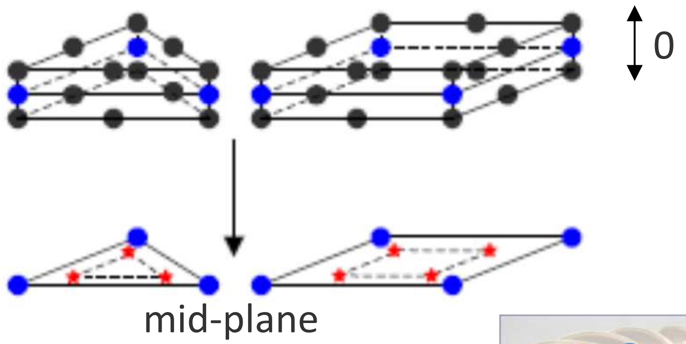
global equilibrium ($\min_{\mathbf{u}} \max_{\boldsymbol{\lambda}}$) — constitutive relation ($\min_{\boldsymbol{\delta}}$)

Spatial discretisation

Nodal Quadratic displacements **u**

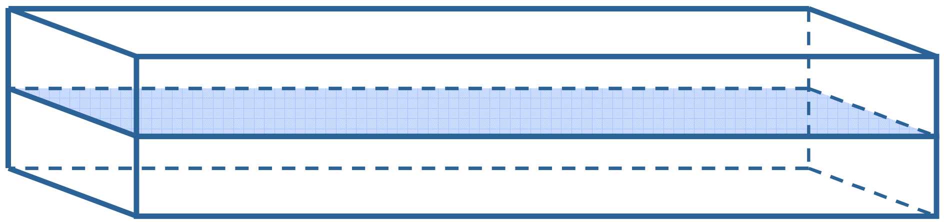
Nodal Linear cohesive forces λ

Gauss points sampled displacement jump δ

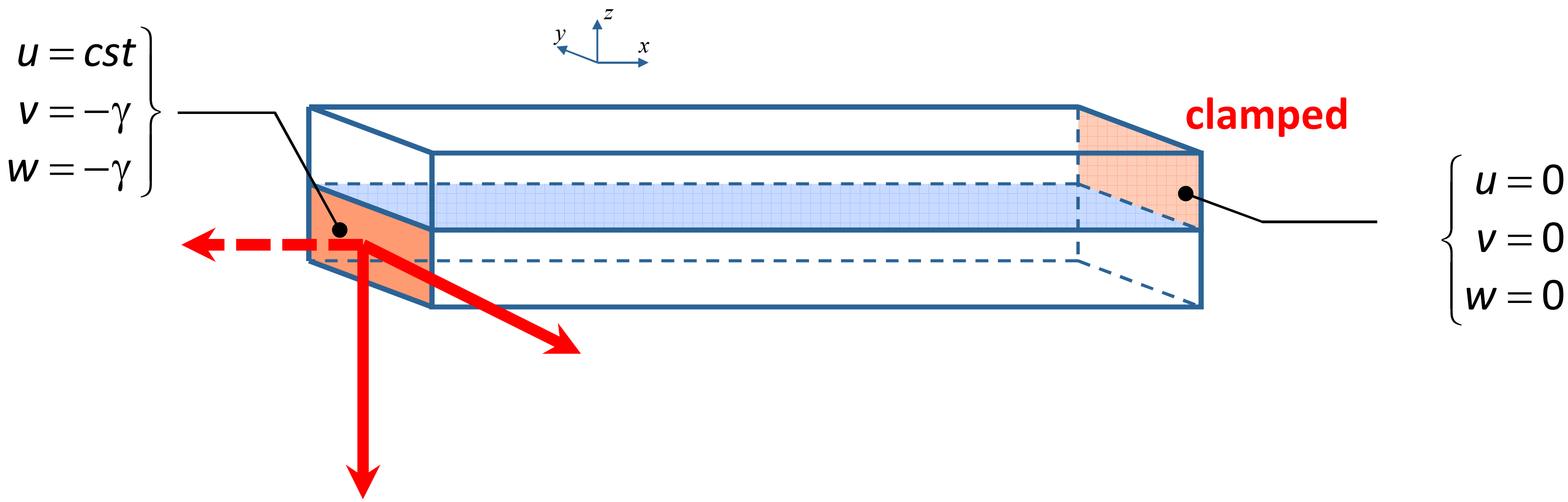


3D SANDWICH BEAMS WITH PRESCRIBED CRACK PATH

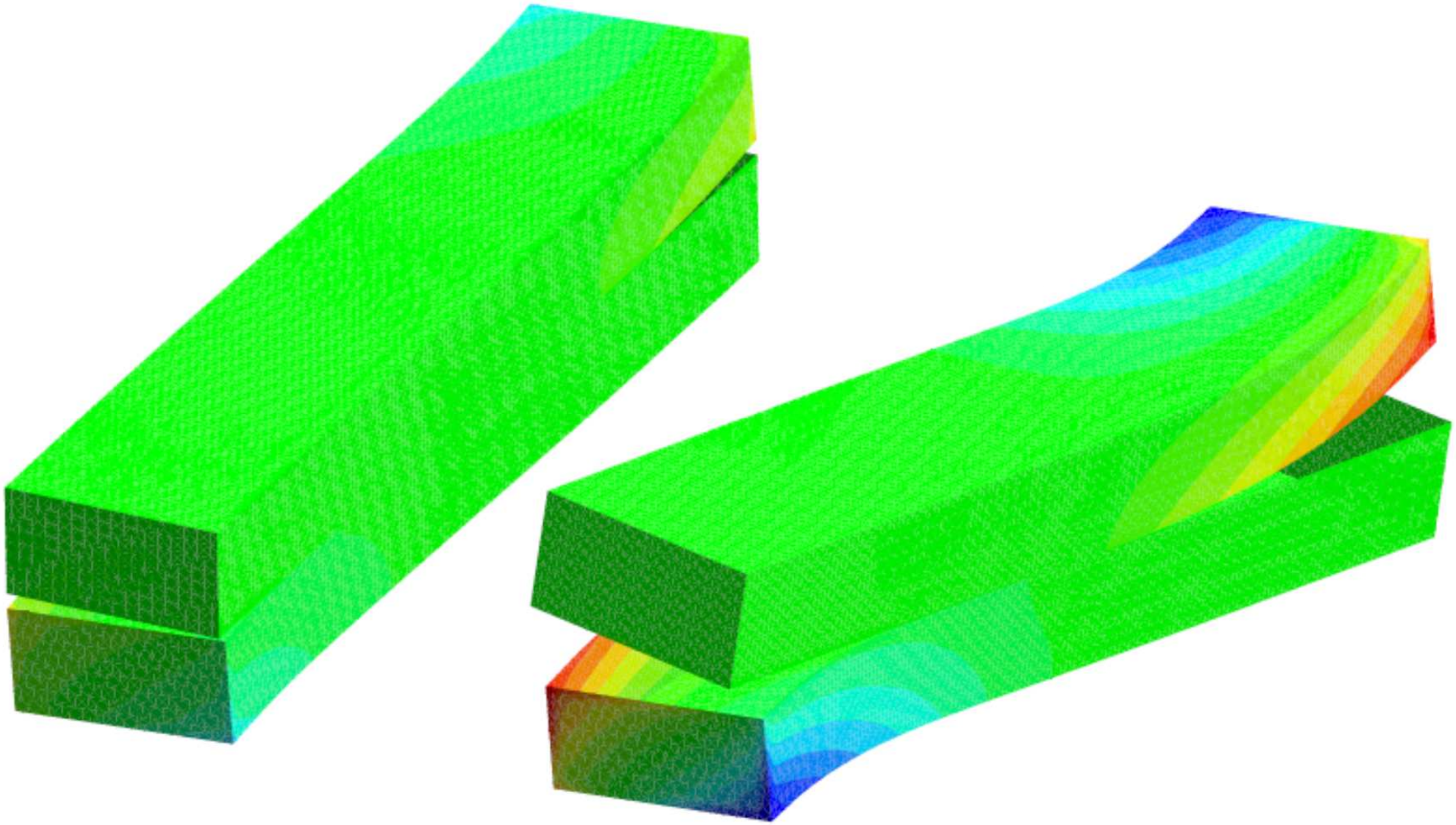
Geometry and crack path



Loading and boundary conditions

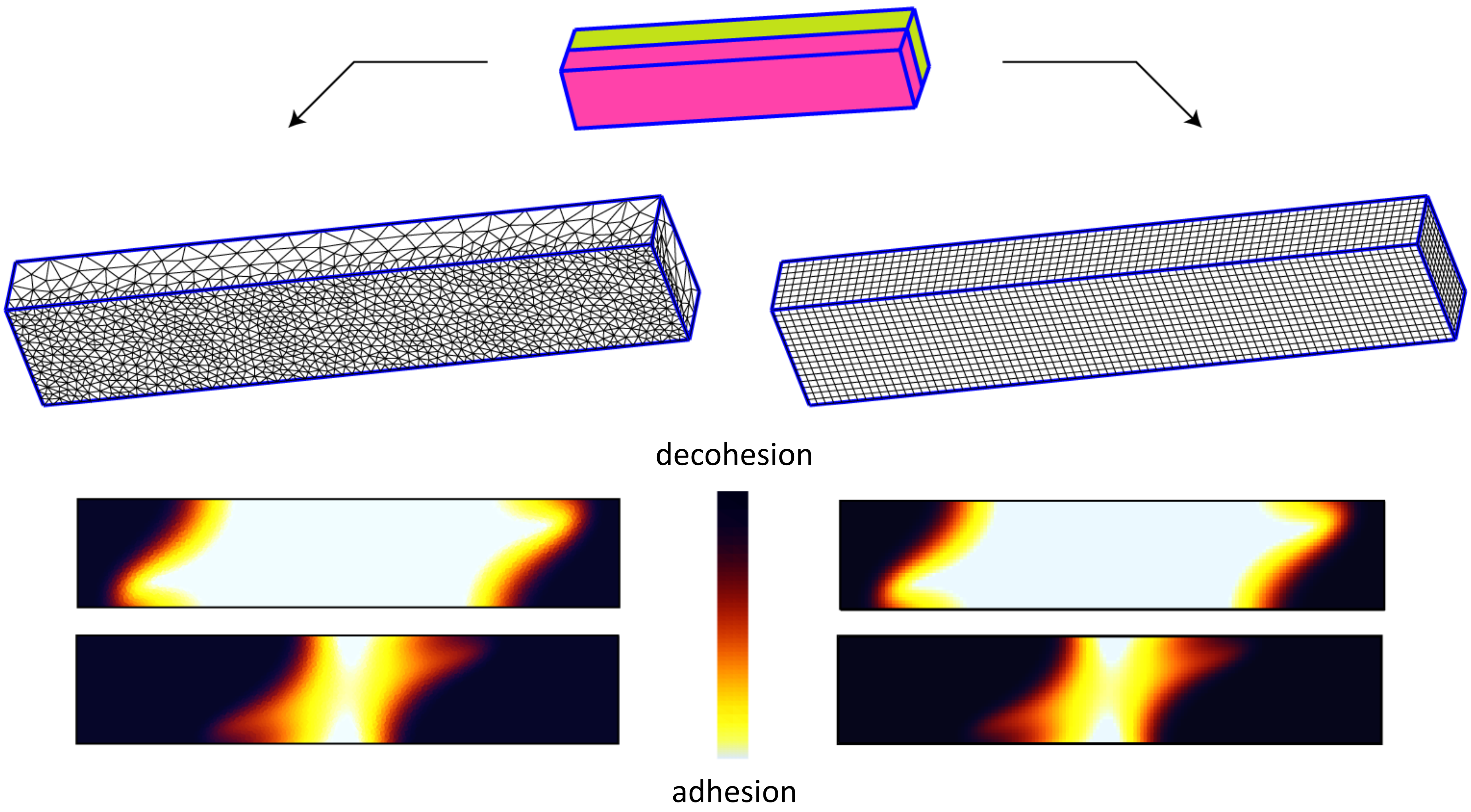


DEFORMED SHAPE AND STRESS FIELD



Deformed shape and longitudinal stress

PROCESS ZONE PROPAGATION



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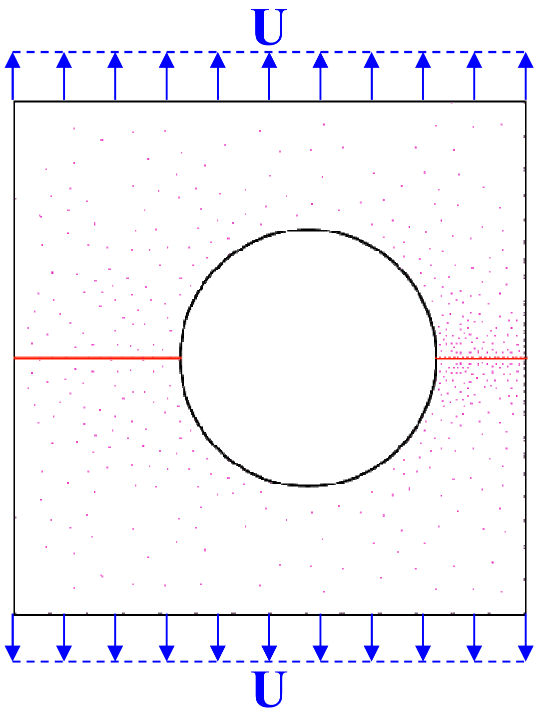
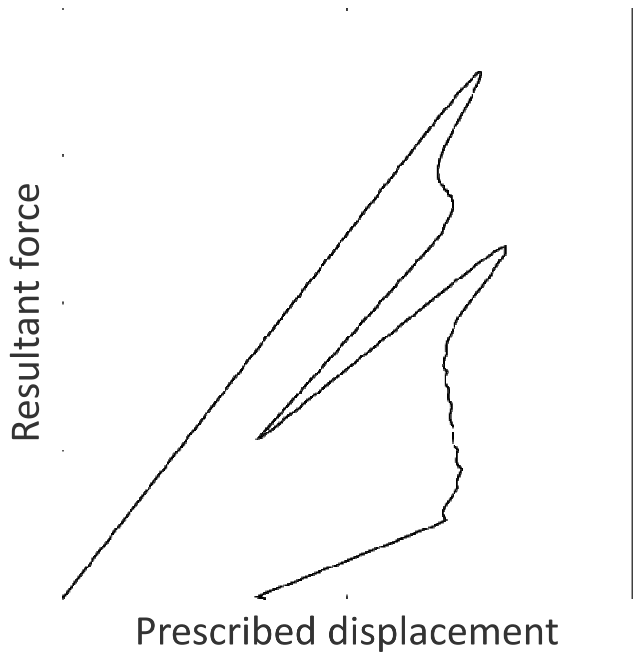
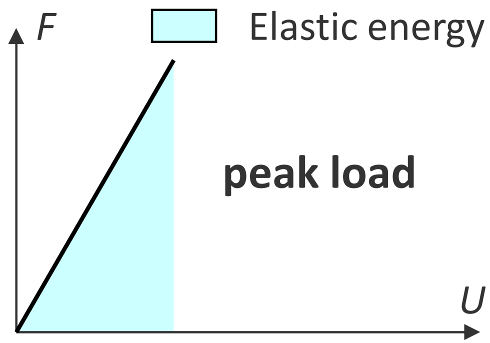
EXEMPLE OF STRUCTURAL INSTABILITIES

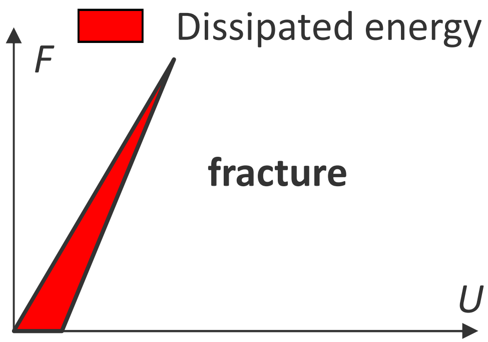




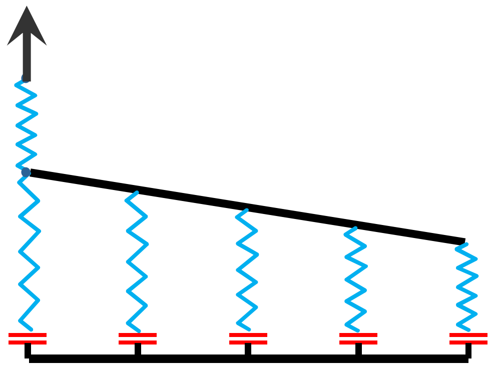
ORIGIN OF THE INSTABILITIES

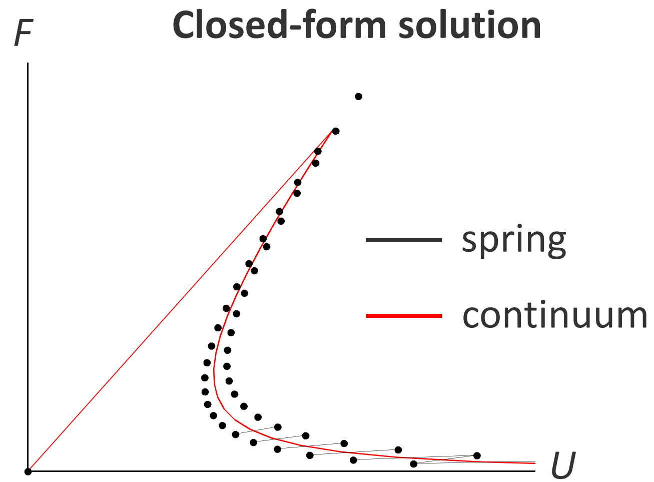
Physical instabilities



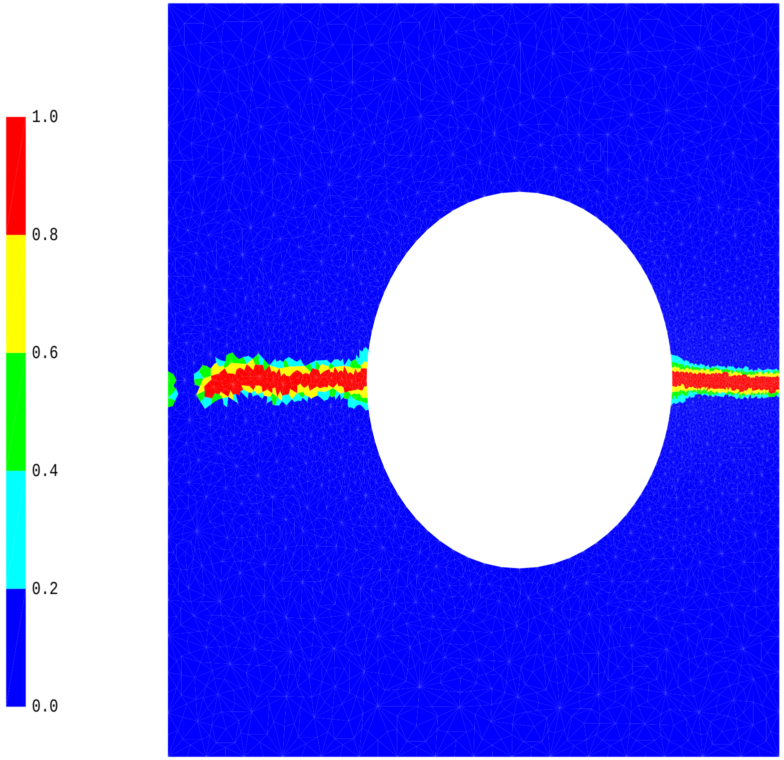


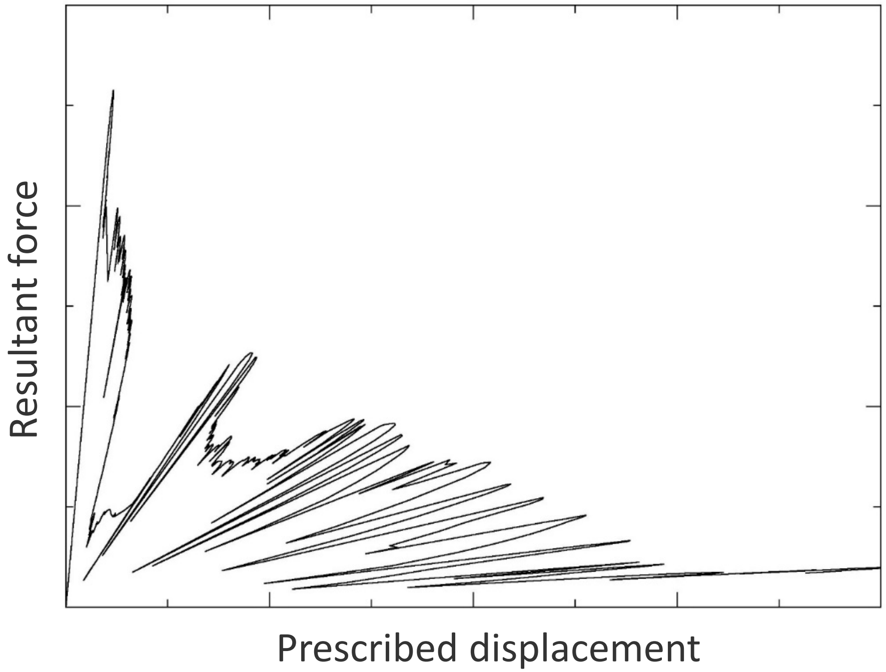
Numerical instabilities





INSTABILITIES ALSO APPEAR WITH CDM





Physical and (questionable) numerical instabilities

PATH-FOLLOWING TECHNIQUE (ARC-LENGTH)

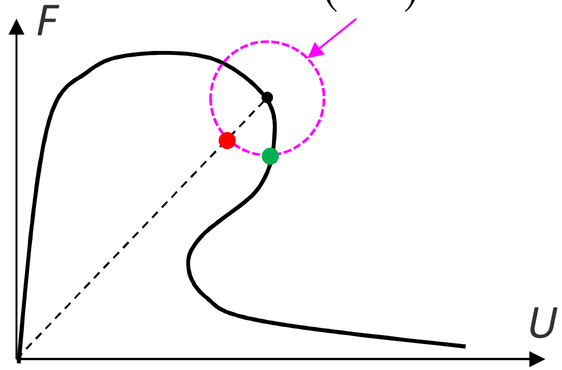
Idea : unknown load amplitude η

$$\mathbf{F}_{\text{int}}(\Delta\mathbf{U}) = t\,\mathbf{F}_{\text{ext}}$$

$$\begin{cases} \mathbf{F}_{\text{int}}(\Delta\mathbf{U}) = \eta\,\mathbf{F}_{\text{ext}} \\ \mathrm{P}(\Delta\mathbf{U}) = \Delta t \end{cases}$$

Role of the path-following function P

$$\mathrm{P}(\Delta\mathbf{U}) = \Delta t$$



Propositions

- Norm of the displacement increment (arc-length)
- Displacement increment in a well-chosen area
- Dissipation increment
- Maximal increment of cohesive damage

$$\mathrm{P}(\Delta\mathbf{U}) = \max_{g \in \text{Gauss}} \left[\Delta\delta_g(\Delta\mathbf{U})\right]$$

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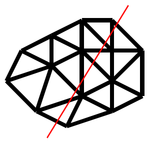
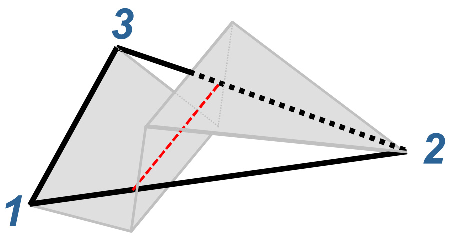
COPING WITH DISCONTINUOUS DISPLACEMENT FIELDS

Mesh independent of Γ

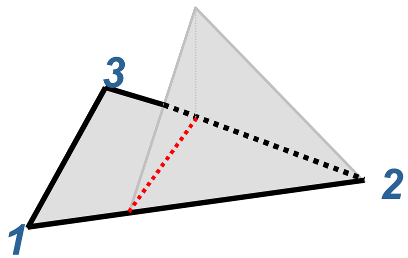


Special finite elements

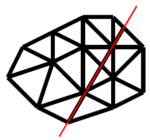
E-FEM
Internal unknowns



X-FEM
nodal unknwowns



Γ along mesh faces



Mesh adaptivity

Remeshing

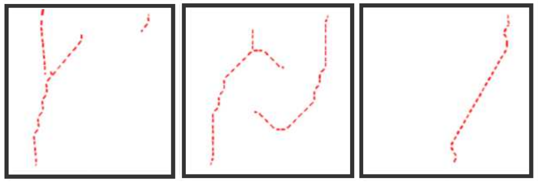
Moving nodes

Mesh quality ?
Field projection ?

ISSUES WITH CRACK PATH PREDICTION

Following all element faces

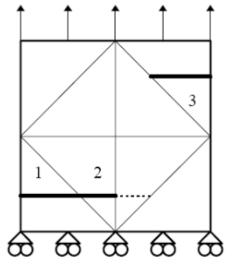
- Extrinsic laws only
- Approximating a curve with fixed segments



Mesh dependency, Feyel (2005)

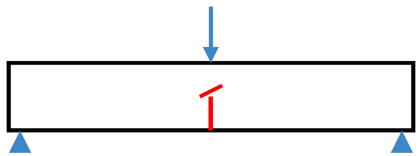
Ensuring crack path continuity

- In order to compute the correct dissipation
- Difficult to ensure step by step 3D continuity



Local crack orientation criterion

- Based on possibly perturbated quantities
- Set earlier or at damage inception (fixed crack)
- What definition in 3D ?



Jirasek & Zimmermann (2005)

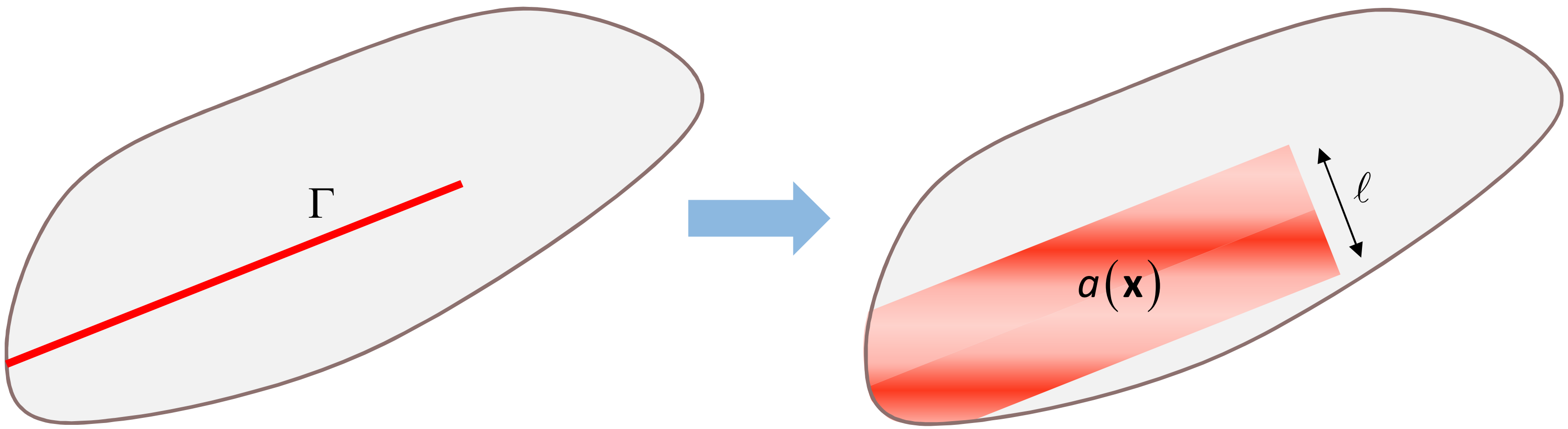
ENERGETIC FORMULATION AND PHASE-FIELD REGULARISATION

Energetic formulation

$$\min_{\mathbf{u}} \left[\ \mathcal{E}_{fr}\left(\llbracket \mathbf{u} \rrbracket\right) + \mathcal{E}_{el}\left(\mathbf{u}\right) - \mathcal{W}_{ext}\left(\mathbf{u}\right) \ \right]$$

Initiation
Cohesive law
Crack path

Phase-field regularisation (Griffith case)



$$\min_{\mathbf{u}} \mathcal{E}\left(\mathbf{u}\right) \quad \rightarrow \quad \mathbf{u}^*$$

$$\min_{\mathbf{u},a} \mathcal{E}_\ell\left(\mathbf{u},a\right) \quad \rightarrow \quad \left(\mathbf{u}_\ell, a_\ell\right)$$

$$\mathbf{u}^* = \lim_{\ell \rightarrow 0} \mathbf{u}_\ell$$

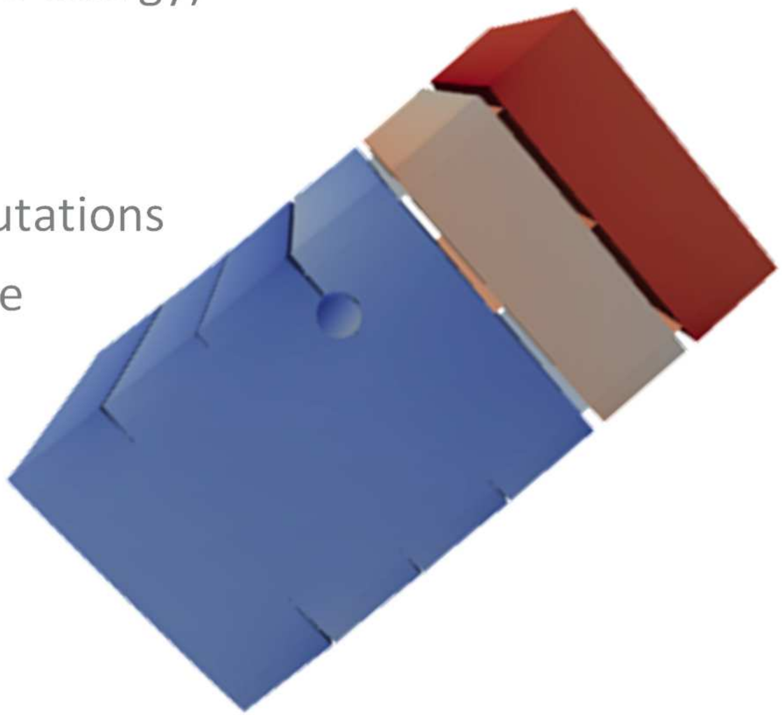
SUMMARY – COHESIVE ZONE MODELS

Strong points

- Deal with initiation, propagation and ultimate failure
- Consistent with Fracture Mechanics
- Realistic crack description (length, opening, …)
- Engineer parameters (peak stress, fracture energy)

Shortcomings

- Nonlinear and potentially unstable computations
- Mesh-refinement inside the cohesive zone
- 3D crack path prediction



Technical tools

- Mixed finite elements
- Path-following methods


Continuum Damage Mechanics

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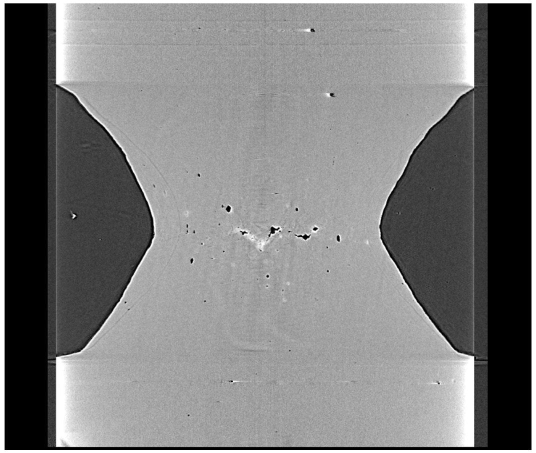
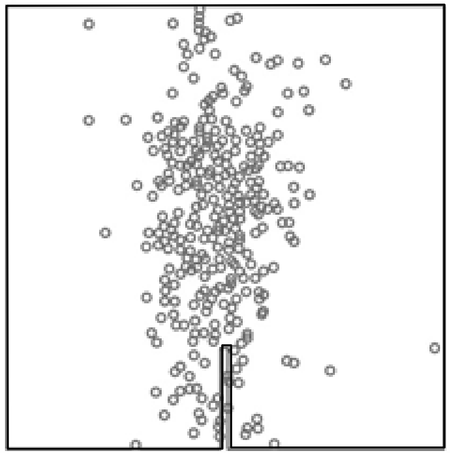
WHY DAMAGE CONSTITUTIVE LAWS ARE NONLOCAL

Damage evolves in layers of small thickness



X100 pipeline steel NT, cavity growth
(Besson, Morgeneyer)



Concrete SENB, acoustic emission energy
Muralidhara et al. (2010)

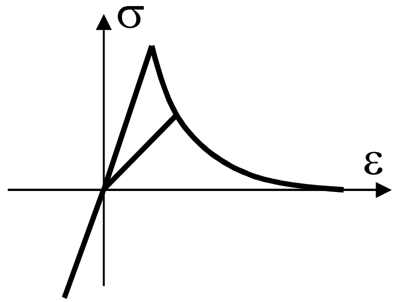
Constitutive relations are nonlocal

- The scale of the damage pattern is comparable to the microstructure size
- The scale separation assumption (homogenisation) does not hold anymore
- A constitutive coupling between neighbour material points takes place

STRAIN-SOFTENING AND DAMAGE LOCALISATION

Strain-softening

The set of admissible stresses shrinks with increasing damage (and strain)



Localisation

Strain-softening and equilibrium enable damage (and strain) localisation

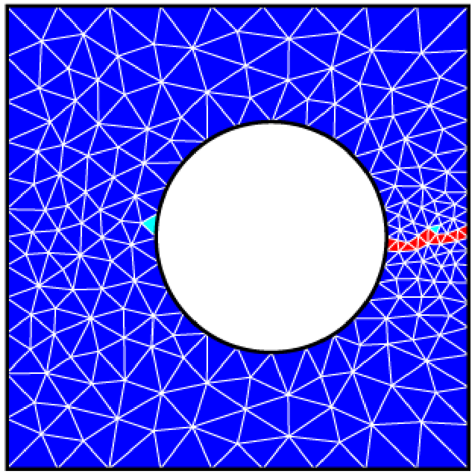
Damage band width

Nonlocality rules the localisation band width

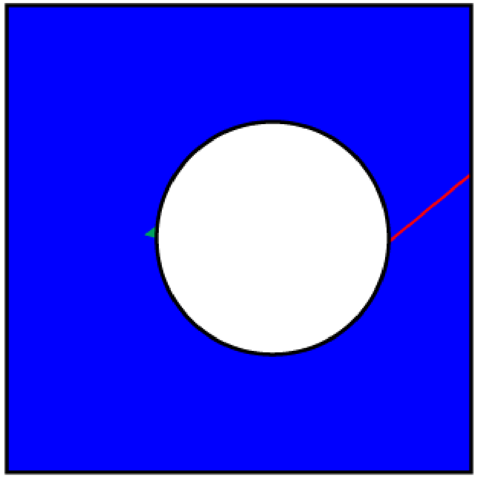
DAMAGE SIMULATIONS WITHOUT NONLOCALITY

Spurious mesh-dependency

Sensitivity to mesh size



Sensitivity to mesh orientation



Ill-posed mathematical problem

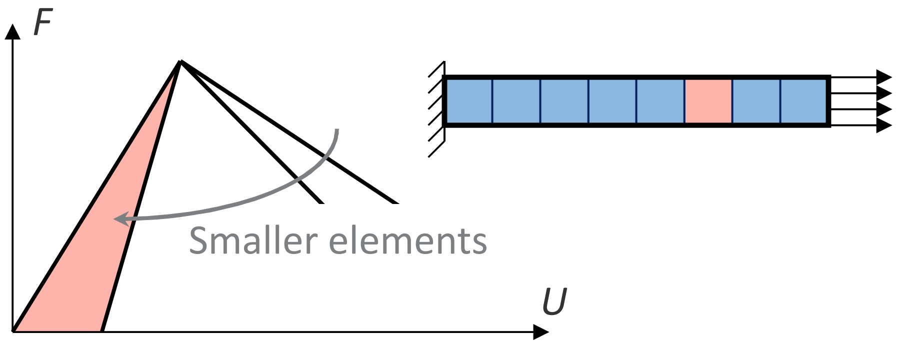
Rate problem

$$\dot{\boldsymbol{\varepsilon}} = \nabla^s \dot{\mathbf{u}} \quad ; \quad \dot{\boldsymbol{\sigma}} = \mathbf{H} : \dot{\boldsymbol{\varepsilon}} \quad ; \quad \operatorname{div} \dot{\boldsymbol{\sigma}} = 0$$

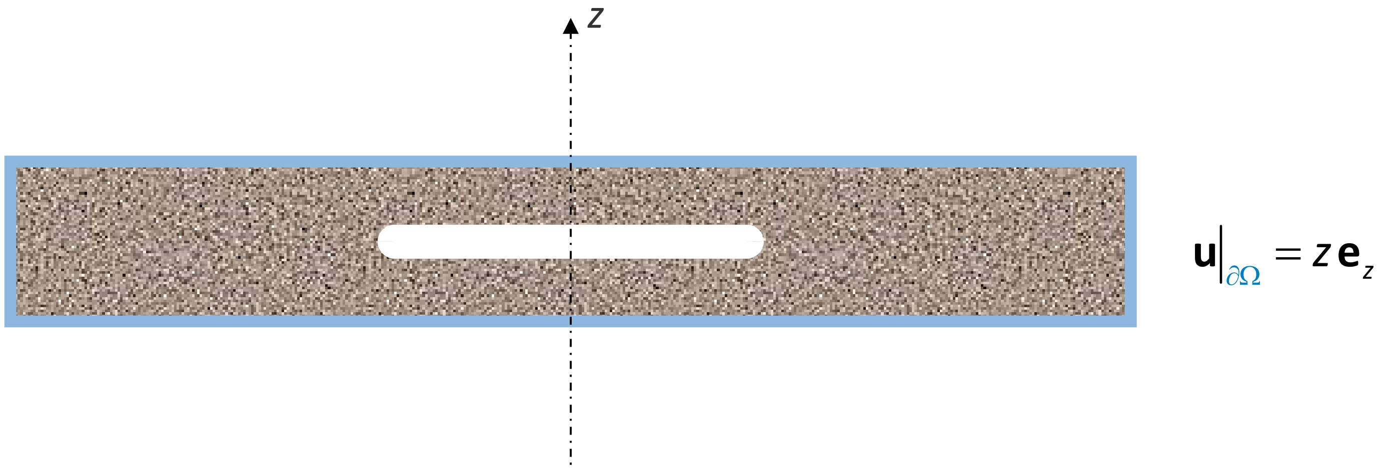
Loss of elllipticity

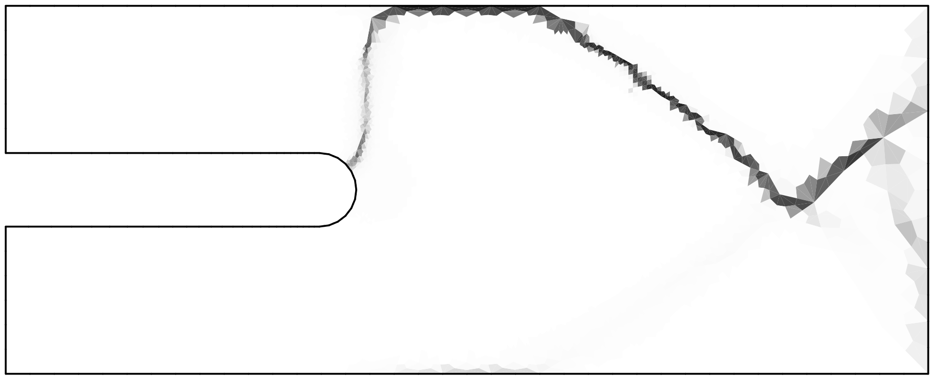
$$\exists \ \mathbf{n} \neq 0 \quad \det(\mathbf{n} \cdot \mathbf{H} \cdot \mathbf{n}) \leq 0$$

Zero dissipated energy



CRACK PATH AND INTUITIVE MESHING





OUTLINE

Cohesive Zone Models (CZM)

- Principle and basic finite elements
- Extrinsic and intrinsic laws – Numerical consequences
- Interface mixed finite elements
- Instabilities and path-following methods
- Crack path prediction and related issues

Continuum Damage Mechanics (CDM)

- Strain-softening, localisation and constitutive nonlocality
- **Nonlocal constitutive relations**
- Gradient models: formulation and mixed finite elements
- Anisotropy vs isotropy and damage – stiffness coupling
- Vanishing internal length and the cohesive limit

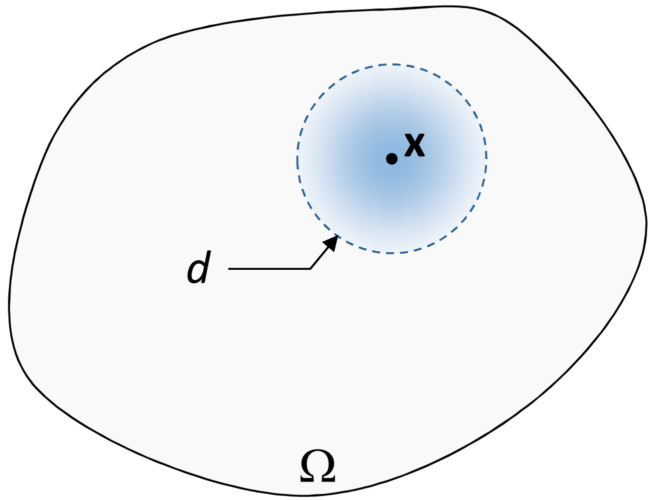
INTRODUCING NONLOCALITY

Principle

- Stress and/or damage depend on what happens elsewhere

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathcal{L}_{\sigma}\left(\boldsymbol{\varepsilon}(\mathbf{y}), a(\mathbf{y})\,;\ \mathbf{y} \in \Omega\right)$$

$$\dot{a}(\mathbf{x}) = \mathcal{L}_{a}\left(\boldsymbol{\varepsilon}(\mathbf{y}), a(\mathbf{y}), \dot{\boldsymbol{\varepsilon}}(\mathbf{y});\ \mathbf{y} \in \Omega\right)$$



Internal length

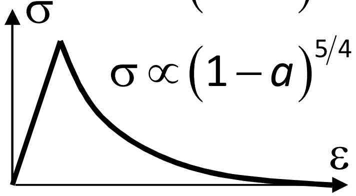
- The influence decreases with distance
- Dimensional analysis : existence of one or several internal lengths *d*

CAN WE SAY THAT NONLOCAL = LOCAL + REGULARISATION ?

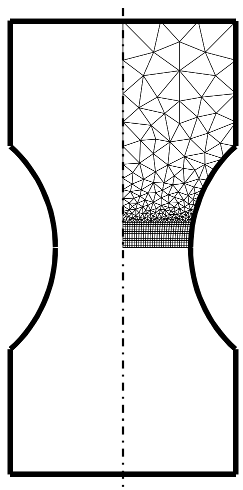
Local constitutive relation

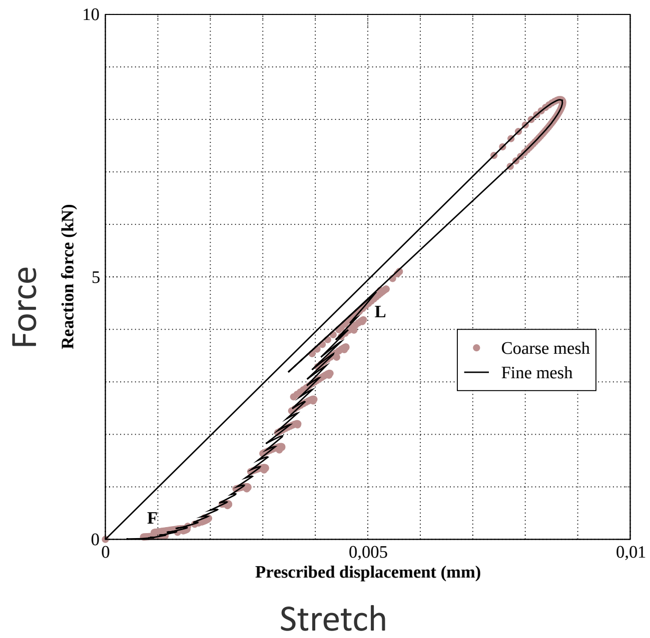
$$\varepsilon \propto (1-a)^{-1/4} \rightarrow \infty$$

$$\sigma \propto (1-a)^{5/4} \rightarrow 0$$

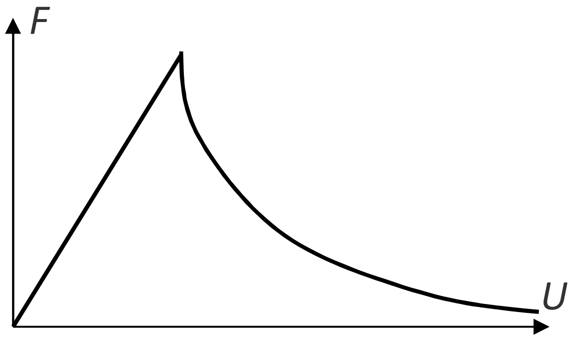


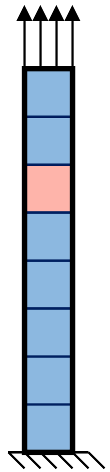
NT specimen



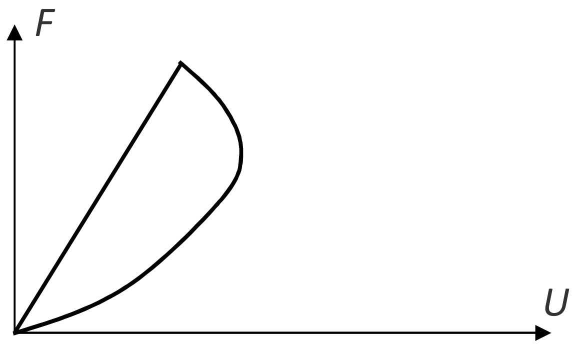


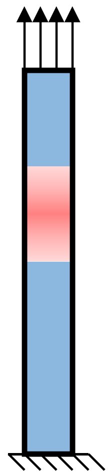
Local + mesh size





Local + gradient damage





ENERGETIC APPROACH

State variables

Kinematics $\mathbf{u}, \boldsymbol{\varepsilon}(\mathbf{u})$

Damage a

Potential energy

$$\mathcal{E}(\mathbf{u}, a) = \mathcal{F}(\boldsymbol{\varepsilon}, a) - \mathcal{W}_{ext}(\mathbf{u})$$

Minimisation principle

$$\mathbf{u}^* = \underset{\mathbf{u}\in\mathcal{C}}{\operatorname{argmin}}\, \mathcal{E}(\mathbf{u}, a^*)$$ Stress definition and equilibrium

$$a^* = \underset{a\in\mathcal{A}}{\operatorname{argmin}}\, \mathcal{E}(u^*, a)$$ Damage evolution

NONLOCAL FORMULATIONS

Micromorphic models

$$\mathcal{E}(\mathbf{u},a)+c\|\nabla\boldsymbol{\chi}\|^2 + h\|\boldsymbol{\varepsilon}-\boldsymbol{\chi}\|^2$$

penalty → $\mathcal{E}(\mathbf{u},a)+c\|\nabla\boldsymbol{\varepsilon}\|^2$

indep. variables → $\mathcal{E}(\mathbf{u},a)+c\|\nabla a\|^2$

Relation to gradient models

Gradient models

$$\mathcal{E}(\mathbf{u},a)+c\|\nabla\boldsymbol{\varepsilon}\|^2$$

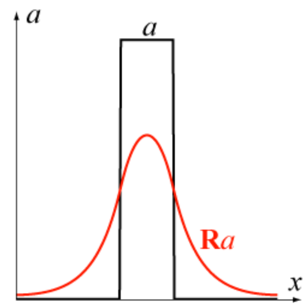
$$\mathcal{E}(\mathbf{u},a)+c\|\nabla a\|^2$$

Penalise high gradients

Localisation limiters

$$\overline{\boldsymbol{\varepsilon}}=\mathbf{R}\boldsymbol{\varepsilon}$$

$$\overline{a}=\mathbf{R}a$$



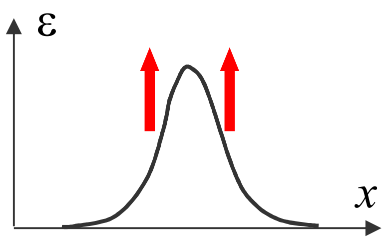
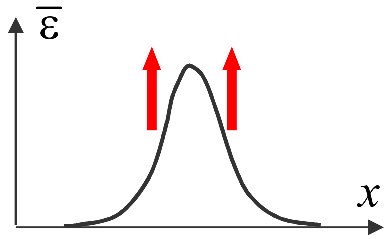
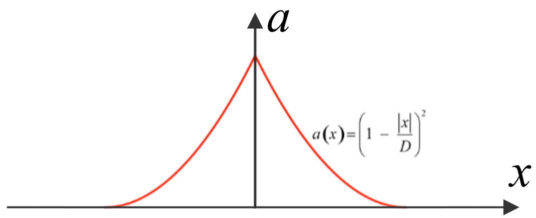
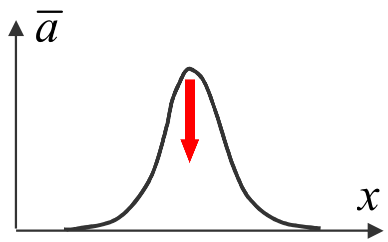
$$\mathcal{F}(\overline{\boldsymbol{\varepsilon}},a) \quad \text{or} \quad \mathcal{F}(\boldsymbol{\varepsilon},\overline{a})$$

1. **Solution existence**
2. **Localisation control**

} **sort**

R defined by an implicit gradient operator (least-square with gradient penalty)

QUALITATIVE ANALYSIS

	Gradient models	**Localisation limiters**
Strain unbounded across a crack	$\boldsymbol{\varepsilon} \in H^1(\Omega)$  Spurious increase of band width	$\bar{\boldsymbol{\varepsilon}} \in H^1(\Omega)$  Spurious increase of band width
Damage Bounded	$a \in H^1(\Omega)$  No a priori limitations	$\bar{a}$ does not reach the bound  Ultimate damage not reached

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STRAIN GRADIENT MODEL AND MIXED FINITE ELEMENTS

Continuum

$$\mathcal{E}(\mathbf{u},a) \equiv \int_\Omega \Phi(\boldsymbol{\varepsilon},a) \; + \frac{c}{2}\|\nabla\boldsymbol{\varepsilon}\|^2 - \mathcal{W}_{ext}(\mathbf{u})$$

$$0 = \delta_{\mathbf{u}}\mathcal{E} = \int_\Omega \underbrace{\left(\frac{\partial\Phi}{\partial\boldsymbol{\varepsilon}} - c\nabla\cdot\nabla\boldsymbol{\varepsilon}\right)}_{\boldsymbol{\sigma}} : \delta\boldsymbol{\varepsilon} \; - \delta\mathcal{W}_{ext} + bnd$$

$$0 = \delta_a\mathcal{E} = \int_\Omega \frac{\partial\Phi}{\partial a}\,\delta a$$

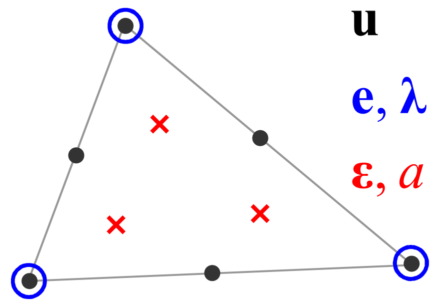
Lagrangian relaxation and spatial discretisation

$$\mathcal{L}(\mathbf{u},a,\mathbf{e},\boldsymbol{\lambda}) \equiv \int_\Omega \Phi(\boldsymbol{\varepsilon},a) \; + \frac{c}{2}\|\nabla\mathbf{e}\|^2 + \boldsymbol{\lambda}:(\boldsymbol{\varepsilon}-\mathbf{e}) + \frac{r}{2}(\boldsymbol{\varepsilon}-\mathbf{e})^2 - \mathcal{W}_{ext}(\mathbf{u})$$

$$0 = \delta_{\mathbf{u}}\mathcal{L} = \int_\Omega \left[\frac{\partial\Phi}{\partial\boldsymbol{\varepsilon}} + \boldsymbol{\lambda} + r(\boldsymbol{\varepsilon}-\mathbf{e})\right] : \delta\boldsymbol{\varepsilon} \; - \delta\mathcal{W}_{ext}$$

$$0 = \delta_{\mathbf{e}}\mathcal{L}(\mathbf{u},a,\mathbf{e},\boldsymbol{\lambda}) = \int_\Omega c\nabla\mathbf{e}:\nabla\delta\mathbf{e} - \boldsymbol{\lambda}:\delta\mathbf{e} - r(\boldsymbol{\varepsilon}-\mathbf{e}):\delta\mathbf{e}$$

$$0 = \delta_{\boldsymbol{\lambda}}\mathcal{L}(\mathbf{u},a,\mathbf{e},\boldsymbol{\lambda}) = \int_\Omega (\boldsymbol{\varepsilon}-\mathbf{e}):\delta\boldsymbol{\lambda}$$



DAMAGE GRADIENT MODEL AND MIXED FINITE ELEMENTS

Continuum

$$\mathcal{E}(\mathbf{u},a) \equiv \int_\Omega \Phi(\boldsymbol{\varepsilon},a) + \frac{c}{2}\|\nabla a\|^2 - \mathcal{W}_{ext}(\mathbf{u})$$

Similar to phase-field methods

$$0 = \delta_{\mathbf{u}}\mathcal{E} = \int_\Omega \frac{\partial \Phi}{\partial \boldsymbol{\varepsilon}} : \delta\boldsymbol{\varepsilon} - \delta\mathcal{W}_{ext}$$

$$0 = \delta_a \mathcal{E} = \int_\Omega \left(\frac{\partial \Phi}{\partial a} - c\nabla^2 a \right) \delta a + bnd$$

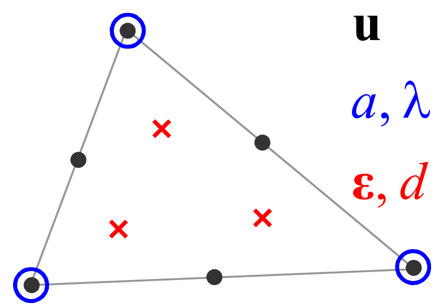
Lagrangian relaxation and spatial discretisation

$$\mathcal{L}(\mathbf{u},a,\lambda,d) \equiv \int_\Omega \Phi(\boldsymbol{\varepsilon},d) + \frac{c}{2}\|\nabla a\|^2 + \lambda(a-d) + \frac{r}{2}(a-d)^2 - \mathcal{W}_{ext}(\mathbf{u})$$

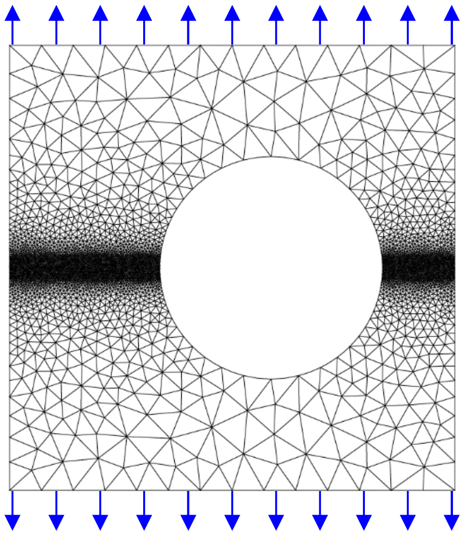
$$0 = \delta_d \mathcal{L}(\mathbf{u},a,\lambda,d) = \int_\Omega \left[\frac{\partial \Phi}{\partial d} - \lambda - r(a-d) \right] \delta d$$

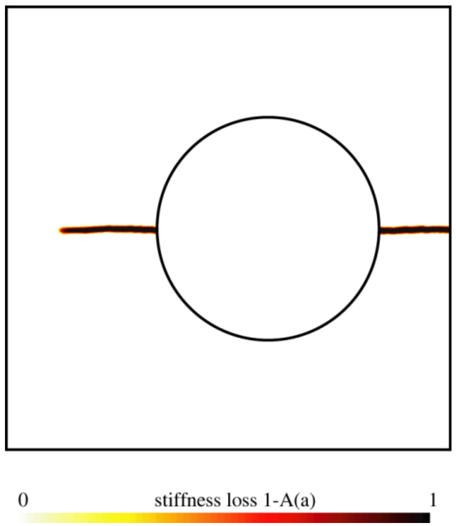
$$0 = \delta_a \mathcal{L}(\mathbf{u},a,\lambda,d) = \int_\Omega c\nabla a : \nabla \delta a + \lambda\, \delta a + r(a-d)\delta a$$

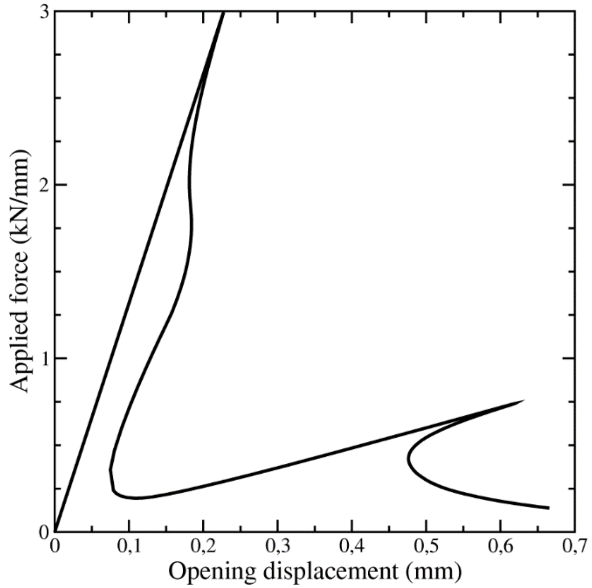
$$0 = \delta_{\boldsymbol{\lambda}} \mathcal{L}(\mathbf{u},a,\lambda,d) = \int_\Omega (a-d)\delta\lambda$$

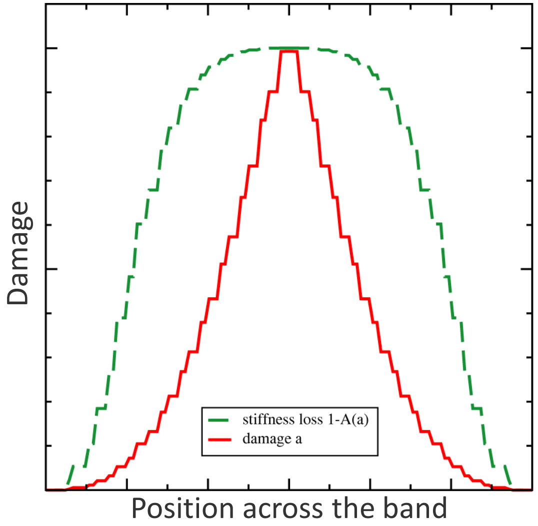


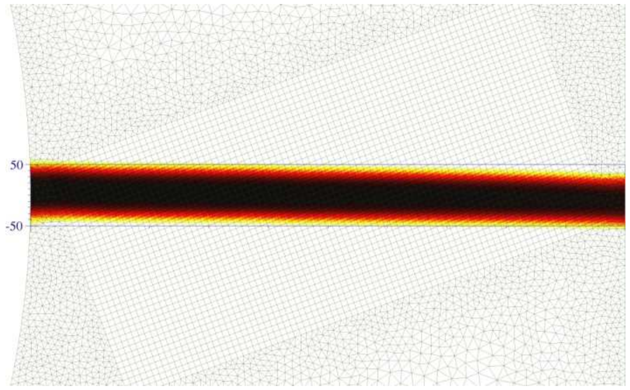
GRADIENT DAMAGE: ROBUSTNESS, RELIABILITY, PERFORMANCE

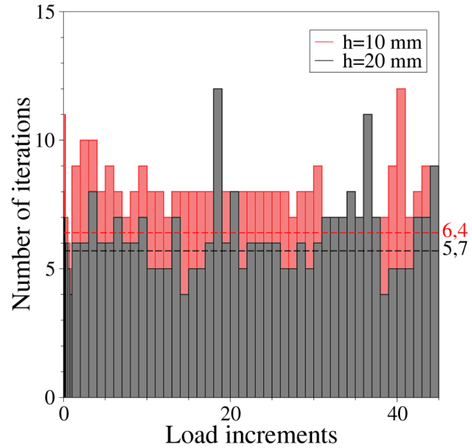




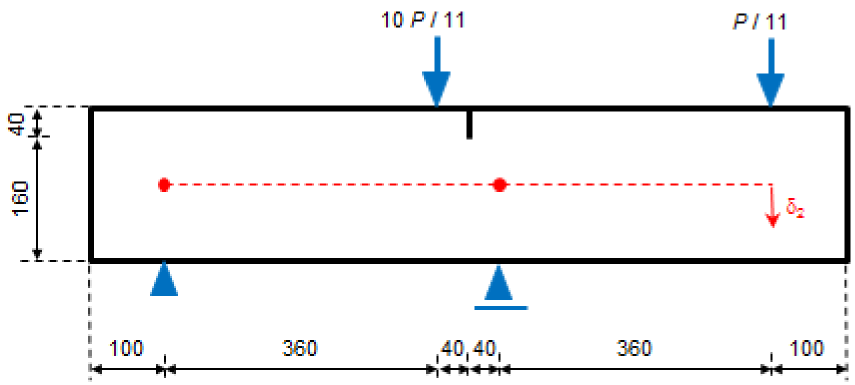


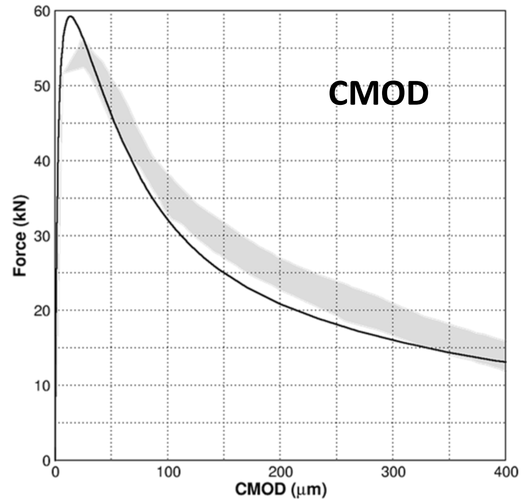


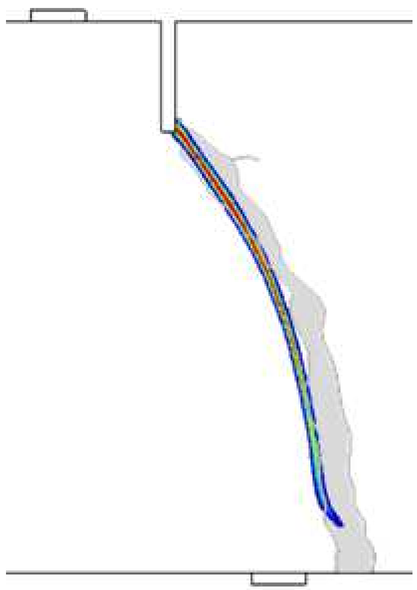




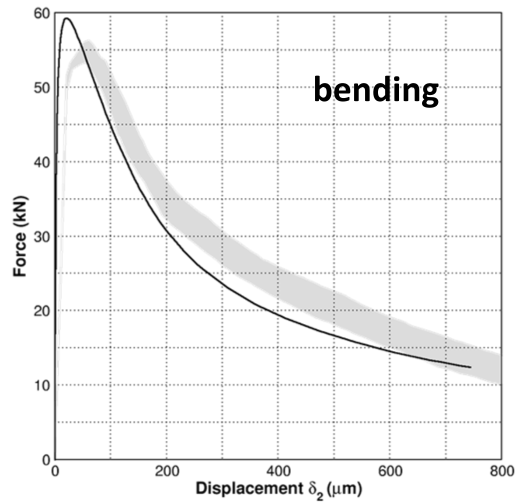
CONCRETE SPECIMEN – UNSYMMETRICAL BENDING



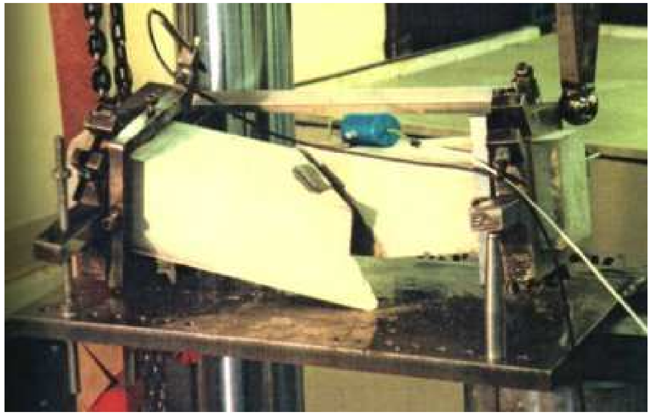


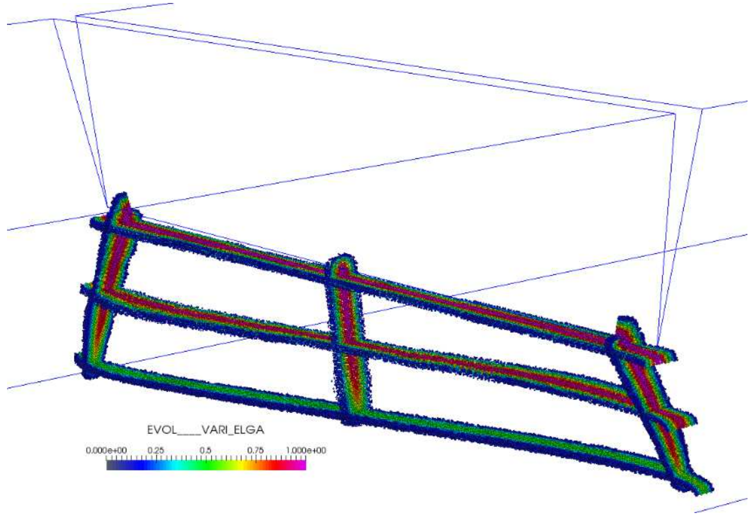


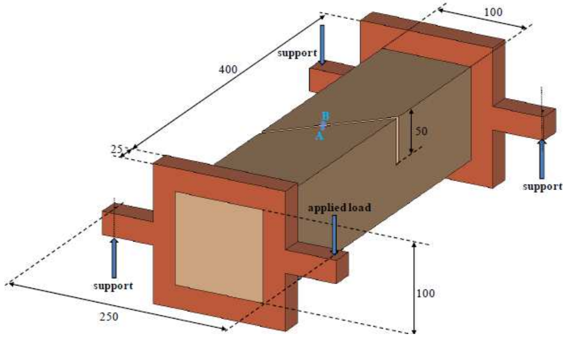
Data calibration on Brazilian and three point bending tests



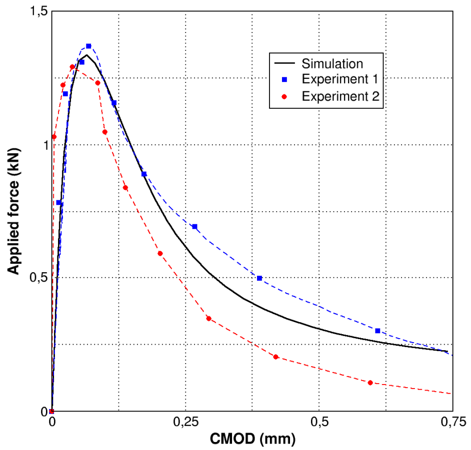
3D CONCRETE SPECIMEN – TORSION LOADING

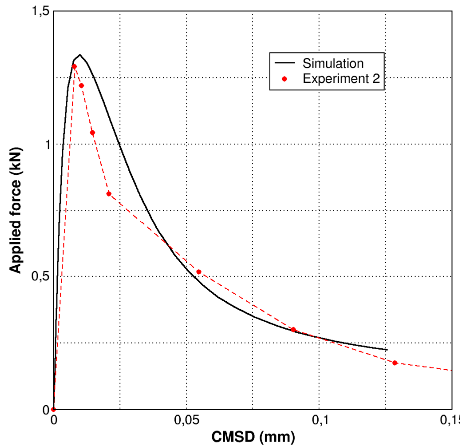




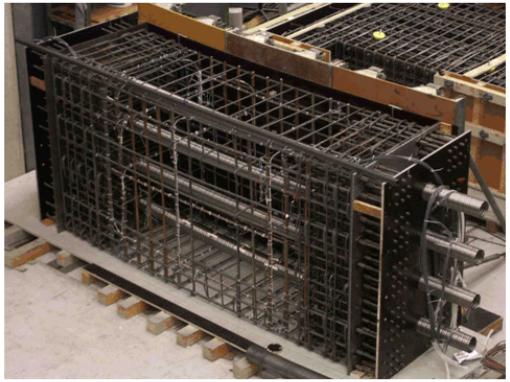


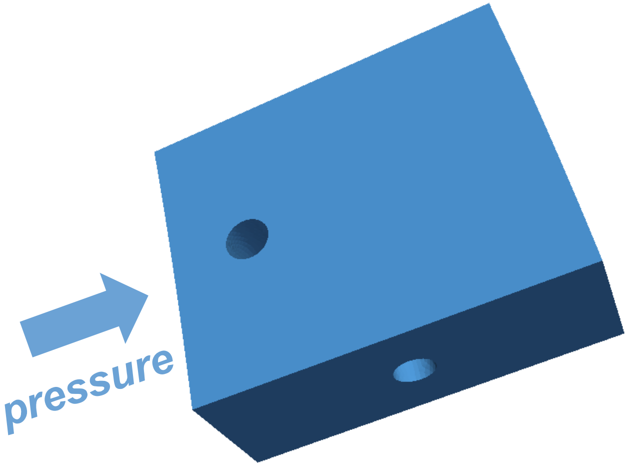
Torsion *10 cm*
Brokenshire (1996)

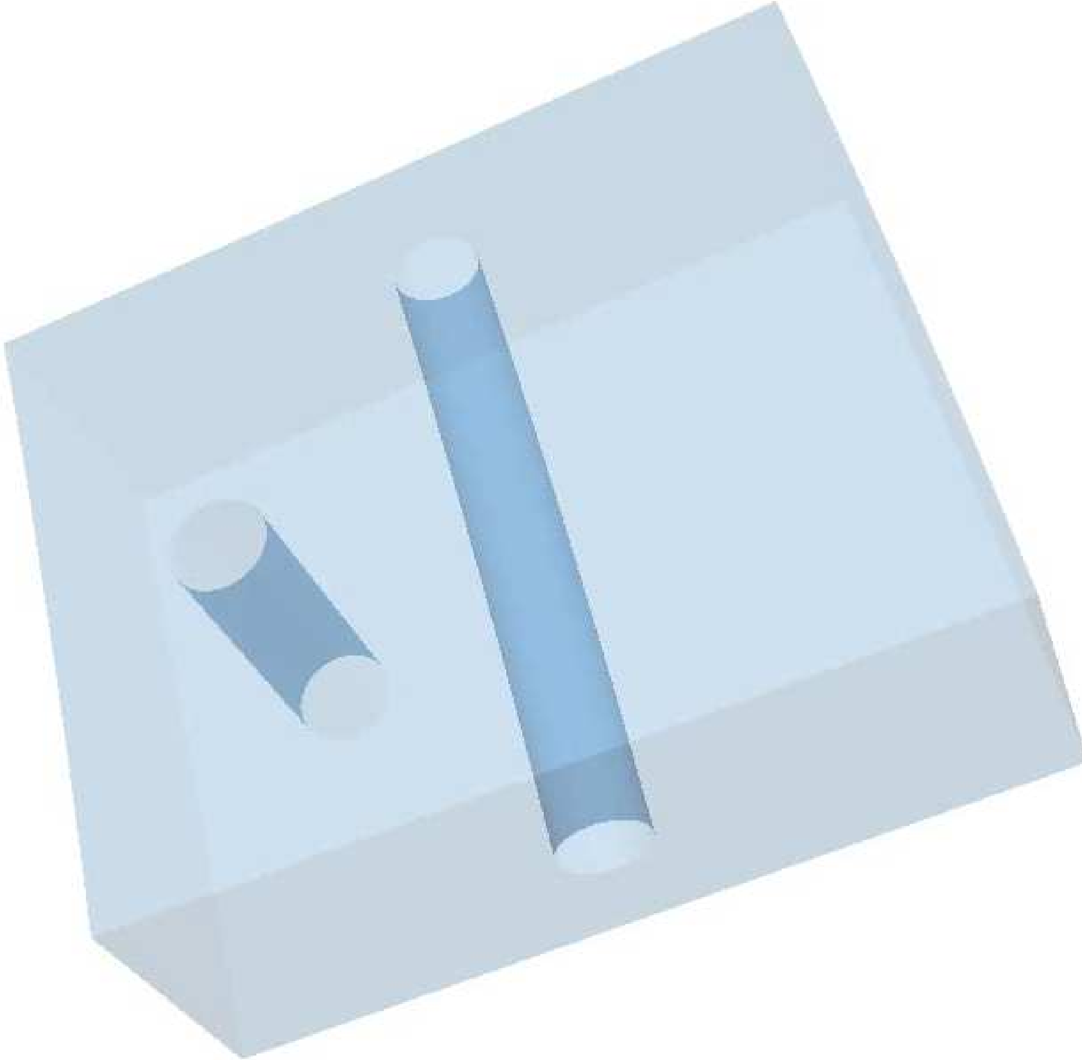




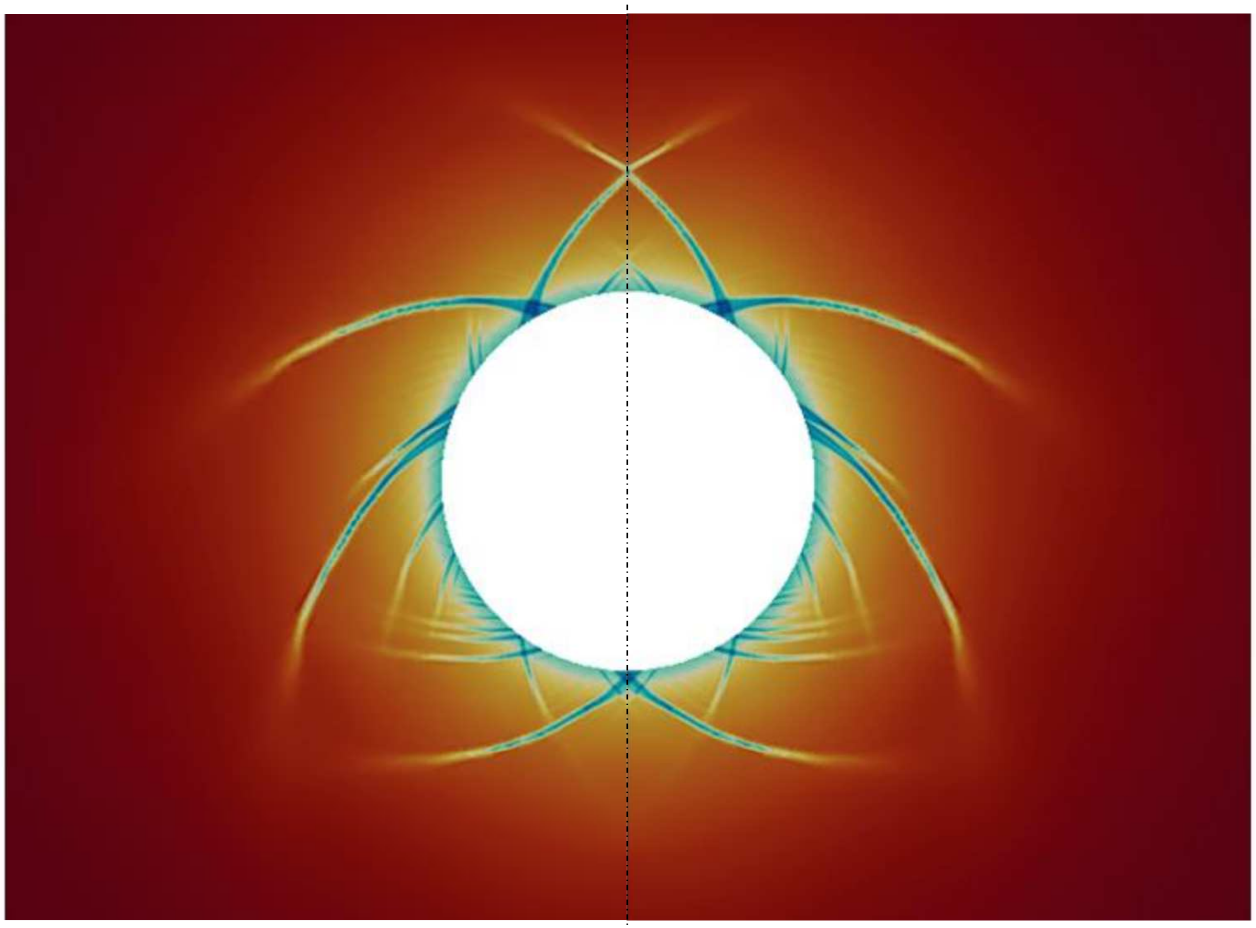
APPLICATION TO A 3D REINFORCED CONCRETE STRUCTURE





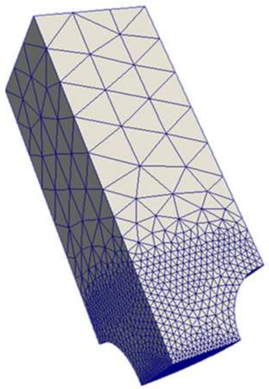


TUNNEL EXCAVATION

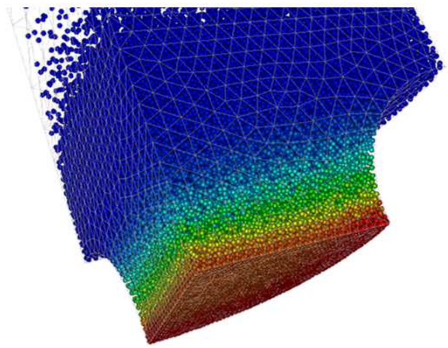


Strain gradient model / strain-softening plasticity

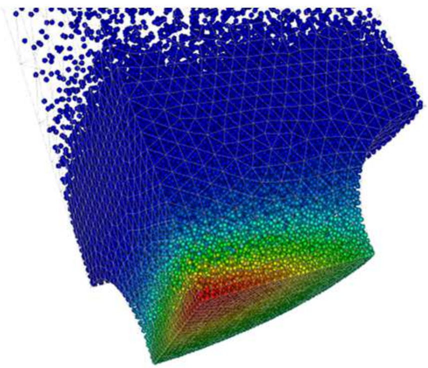
NT DUCTILE SPECIMEN



Mesh



Equivalent plastic strain

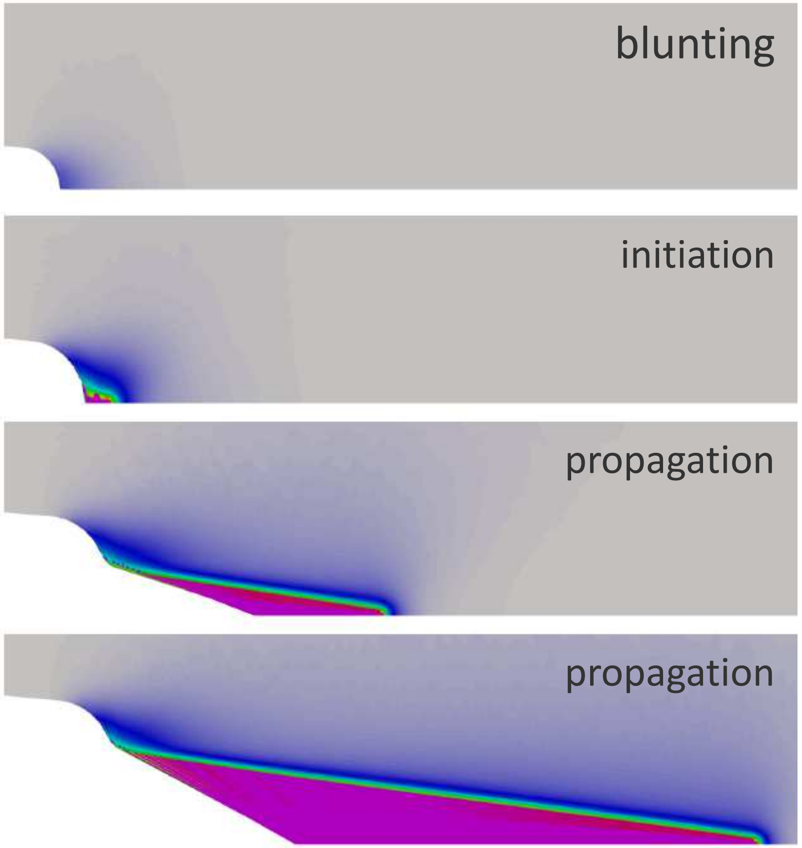


Porosity

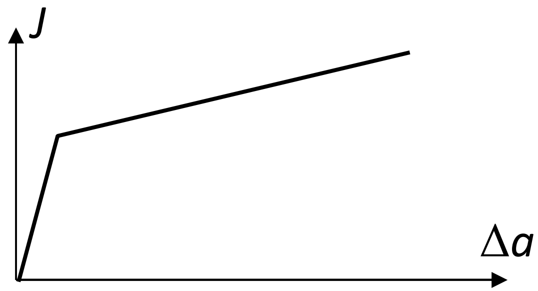
Gradient plasticity – Gurson Tvergaard Needleman (GTN)

DUCTILE FRACTURE NEAR A CRACK TIP

Porosity distribution



Relation to Fracture Mechanics



Constant crack tip opening angle (CTOA)

Critical plasticity during propagation

Gradient plasticity – Gurson Tvergaard Needleman (GTN)

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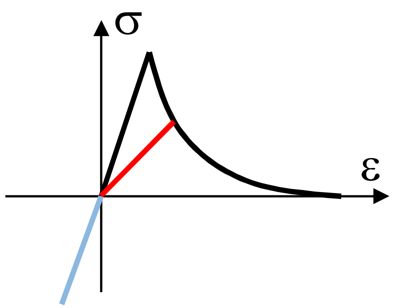
ISOTROPIC OR ANISOTROPIC DAMAGE ?

A question of scale

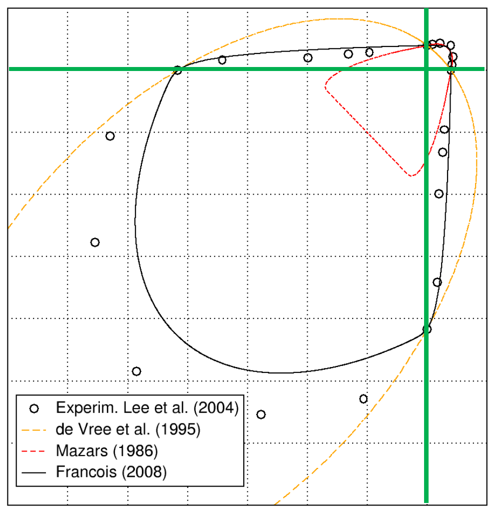
- Homogenised cracks → anisotropic damage
- Single crack → isotropic damage ⇨ anisotropy at higher scale

Isotropy is not contradictory with tension / compression contrast

- On the damage threshold (concrete for instance)
- On the elastic behaviour after damage (crack closure)
- Impact on shear = tension + compression



crack closure



damage surface in stress space

TENSION / COMPRESSION SPLIT

Energy split

$$\frac{1}{2}\boldsymbol{\varepsilon}:\mathbf{E}:\boldsymbol{\varepsilon} = \mathrm{w}^{t}(\boldsymbol{\varepsilon}) + \mathrm{w}^{c}(\boldsymbol{\varepsilon})$$

$$\mathrm{w}(\boldsymbol{\varepsilon}, a) = \mathrm{A}(a)\,\mathrm{w}^{t}(\boldsymbol{\varepsilon}) + \mathrm{w}^{c}(\boldsymbol{\varepsilon})$$

Focus on the “broken” material

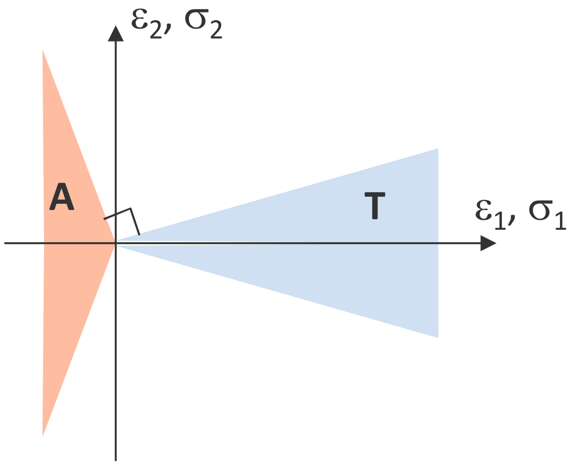
Tensile strain tensors

$$\mathcal{T} = \left\{\boldsymbol{\varepsilon}\,;\ \mathrm{w}^{c}(\boldsymbol{\varepsilon}) = 0\right\}$$

Admissible stress tensors

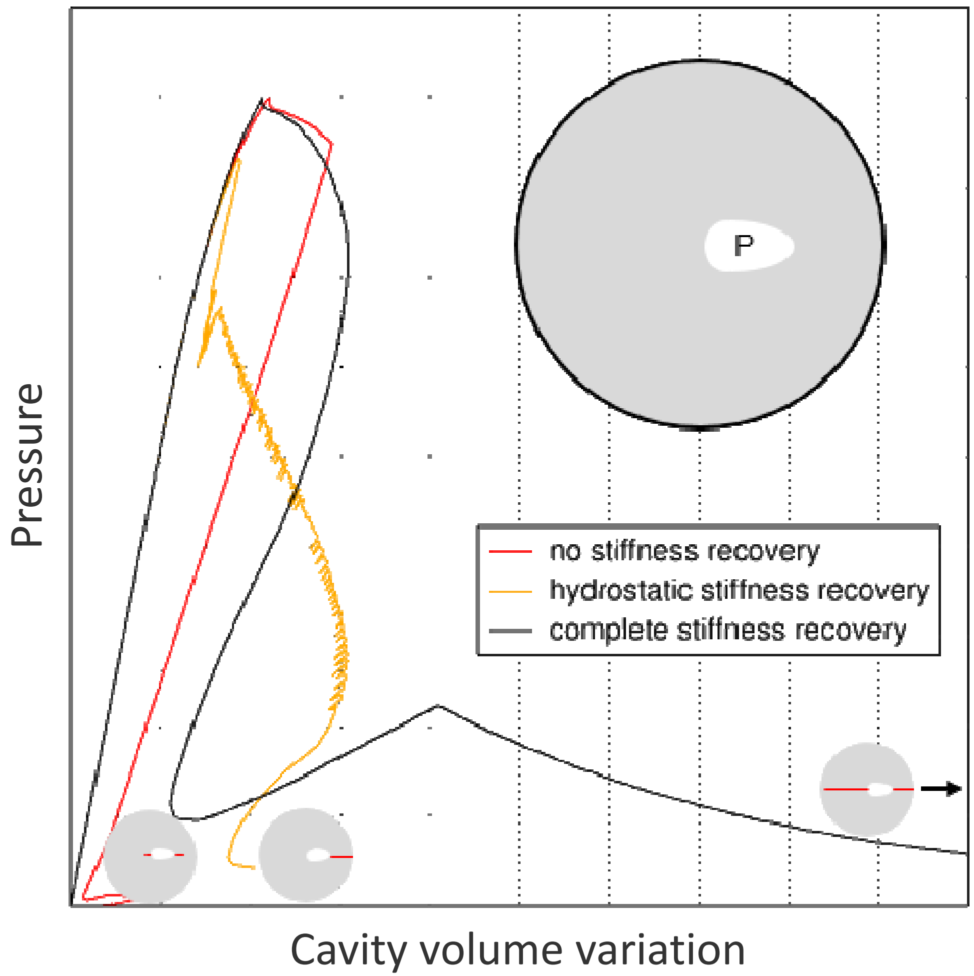
$$\mathcal{A} = \left\{\boldsymbol{\sigma}\,;\ \mathrm{w}^{c^*}(\boldsymbol{\sigma}) < \infty\right\}$$

Property: ***$\mathcal{T}$ and $\mathcal{A}$ are orhtogonal cones***

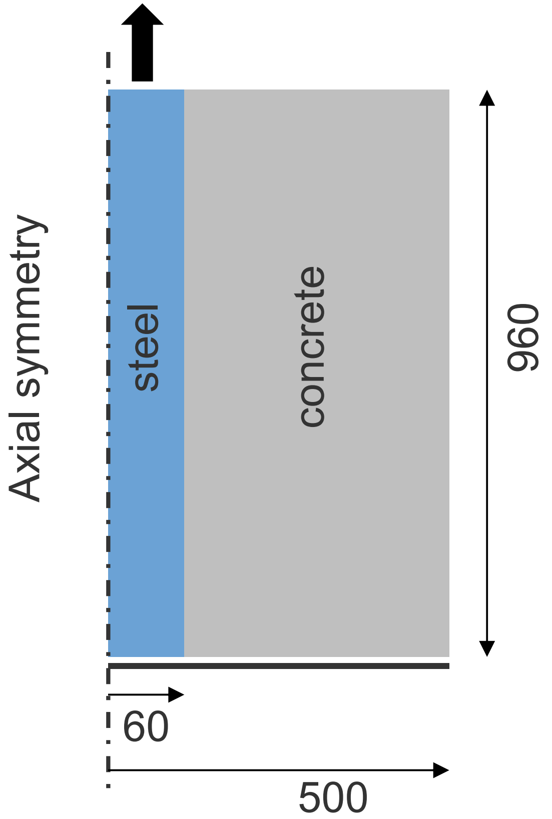


e.g. $\mathcal{T} = \left\{\boldsymbol{\varepsilon}\,;\mathrm{tr}(\boldsymbol{\varepsilon}) \geq 0\right\}$; $\mathcal{A} = \left\{\boldsymbol{\sigma} = -p\mathbf{Id}\,; p \geq 0\right\}$

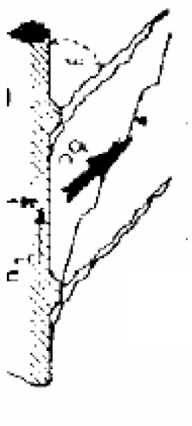
PRACTICAL CONSEQUENCE OF DAMAGE / STIFFNESS COUPLING



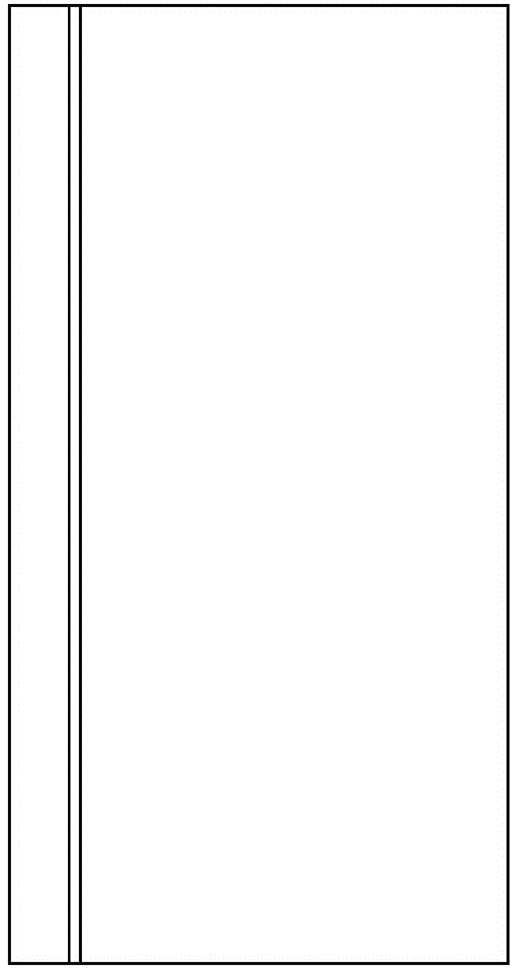
SHEAR DAMAGE NEAR A TENDON (STEEL / CONCRETE INTERFACE)



Experiment



Resistance thanks to
compressive stiffness



OUTLINE

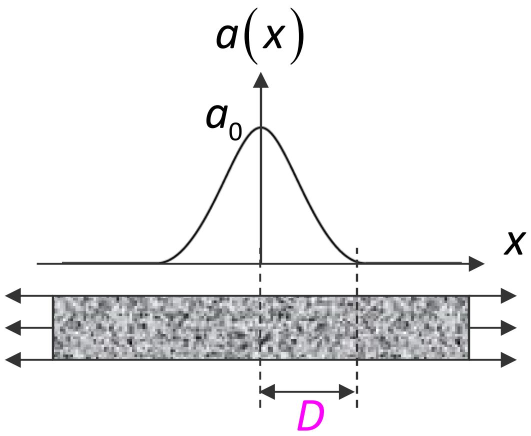
Cohesive Zone Models (CZM)

- Principle and basic finite elements
- Extrinsic and intrinsic laws – Numerical consequences
- Interface mixed finite elements
- Instabilities and path-following methods
- Crack path prediction and related issues

Continuum Damage Mechanics (CDM)

- Strain-softening, localisation and constitutive nonlocality
- Nonlocal constitutive relations
- Gradient models: formulation and mixed finite elements
- Anisotropy vs isotropy and damage – stiffness coupling
- **Vanishing internal length and the cohesive limit**

CONSISTENCY WITH A COHESIVE ZONE MODEL

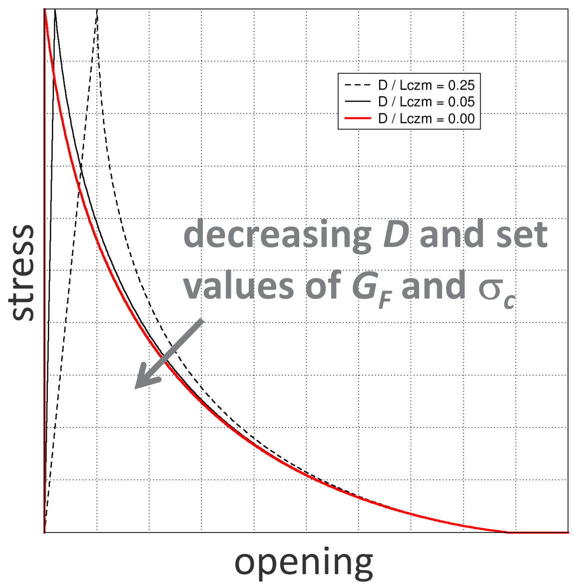


Necessary value for finite opening displacement

Exponent consistent with phase-field model

$$\boldsymbol{\sigma} = A(a)\,\mathbf{E} : \boldsymbol{\varepsilon}$$

$$A(a) = \frac{(1-a)^2}{(1-a)^2 + \frac{3}{2}\frac{E_c G_F}{\sigma_c^2 D} a \bar{A}(a)}$$



Meaningful parameters

- Young modulus
- Peak stress σ_c
- Fracture energy G_F

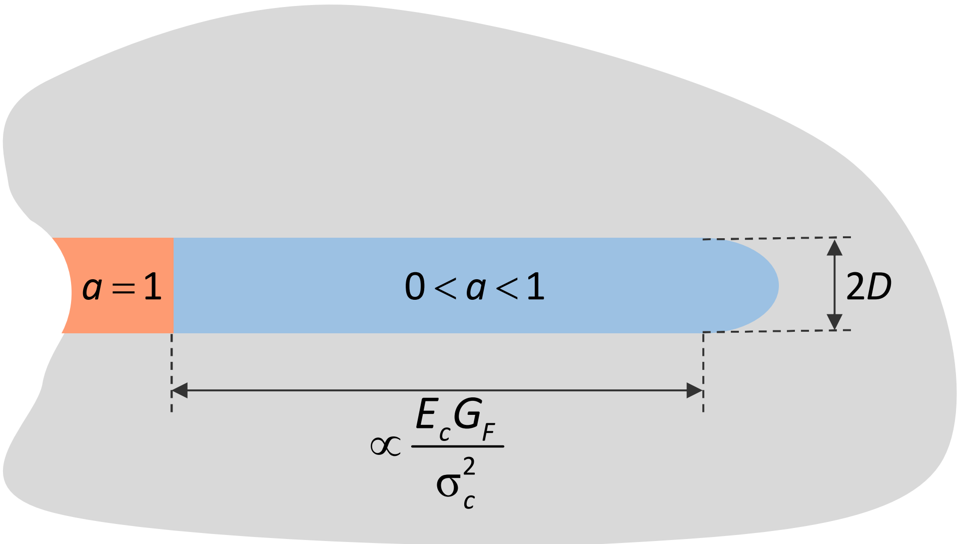
- Band width D

PRACTICAL CHOICE OF THE CHARACTERISTIC LENGTH D

How should the internal length D be calibrated ?

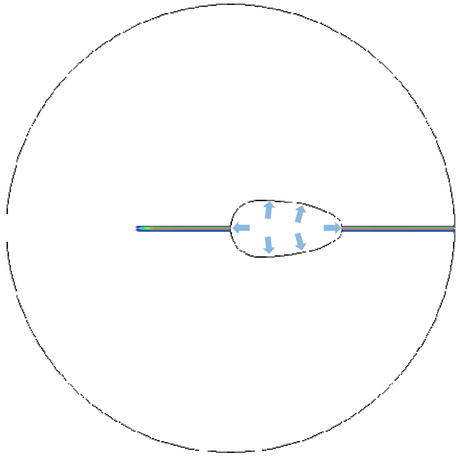
- Interest in the macroscopic response only
- The macroscopic results are not sensitive to (small) values of D

D small compared to the structure (~ *L*/10) and sufficiently large to avoid any numerical burden

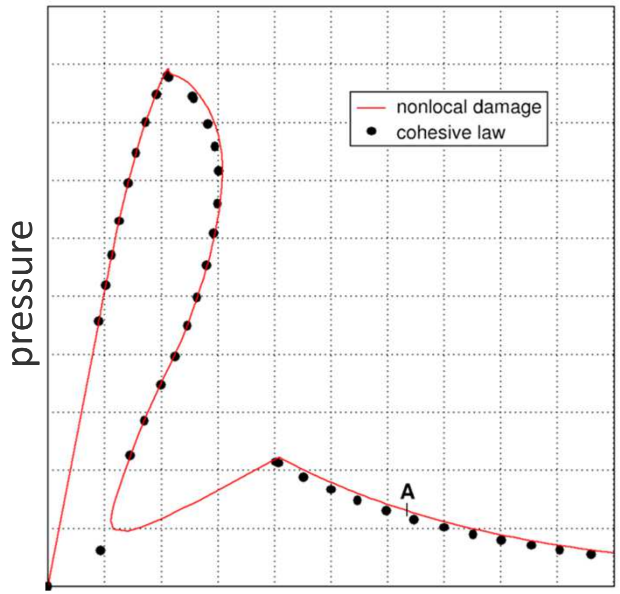


cohesive response + crack path prediction

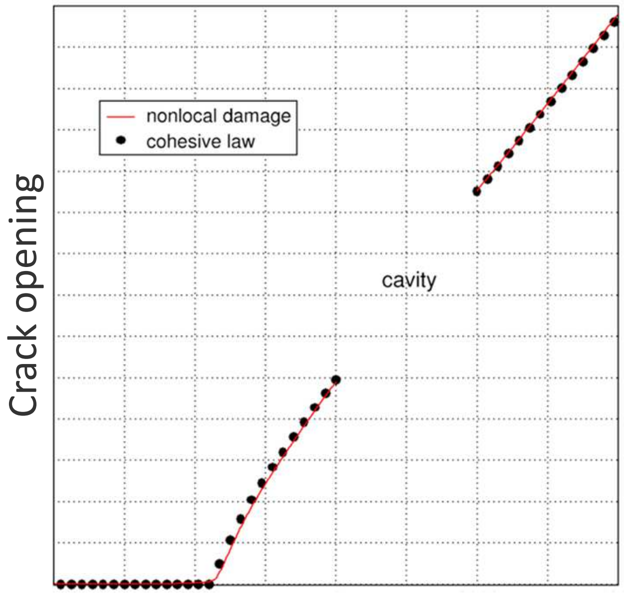
CONSISTENCY BETWEEN CZM AND GRADIENT DAMAGE



Global response



Opening profile



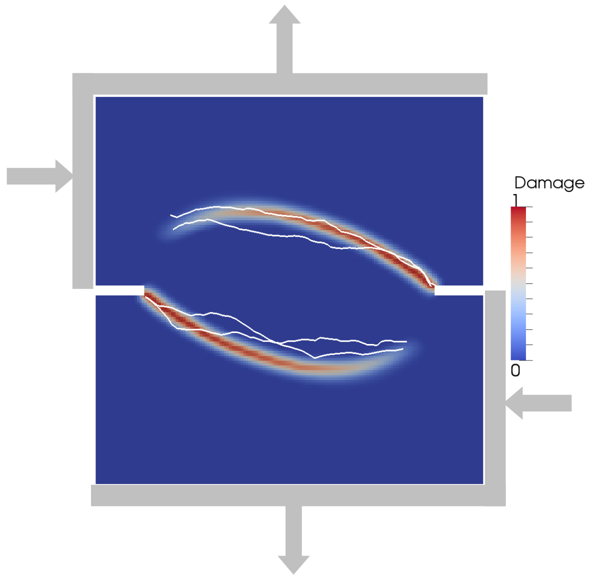
SUMMARY – CONTINUUM DAMAGE MECHANICS

Strong points

- Description of all phases of the damage process
- Crack path prediction

Shortcomings

- Description of a real crack
- Parameter identification
- Highly nonlinear computations
- Highly expensive computations
- Necessary mesh-adaptivity



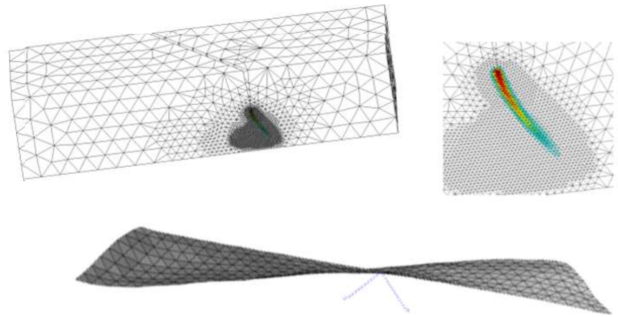
Technical tools

- Nonlocal constitutive relations

WHAT WAS SET ASIDE

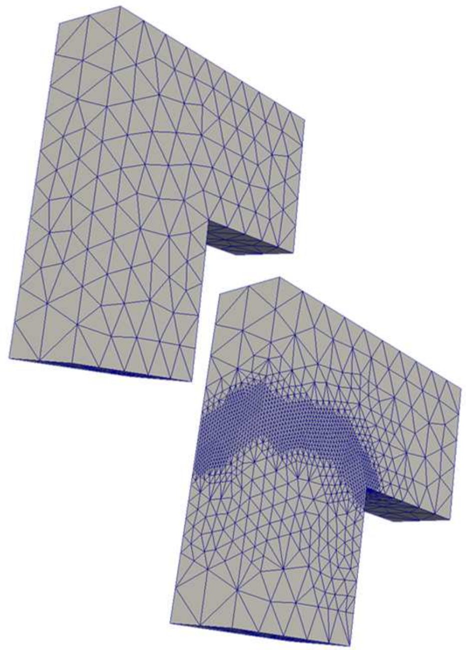
Modelling

- Thick level set (TLS)
- Boundary conditions
- Transition from localised damage to crack



Numerical treatments

- Convergence criteria
- Incompressibility and volumetric locking
- Numerical schemes and solvers



Computation

- Mesh adaptivity

MY PHILOSOPHY REGARDING NUMERICAL STRATEGIES

1. Avoid damage computations if post-treatment criteria are applicable

- Energy release rate *G*, path integral *J*
- Rice and Tracey growth criterion

2. If you can guess potential crack paths, use Cohesive Zone Models

- No stiffness regularisation (extrinsic laws)
- Mixed finite elements along the crack path
- Path-following methods if instabilities are expected

3. Otherwise rely on Continuum Damage Mechanics

- Nonlocal constitutive laws (preferably gradient models)
- Refined mesh in the damaged areas
- High computational cost