Technologie X-FEM pour la modélisation de la rupture : avantages, limitations et utilisation dans le cadre du modèle Thick Level Set (TLS)

Nicolas Moës
Ecole Centrale de Nantes, Institut GeM CNRS
Institut Universitaire de France

CENTRALE NANTES

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Institut de Recherche en Génie Civil et Mécanique

TLS Nantes Collaborators (2011-2019)

- N. Chevaugeon, G. Legrain, L. Stainier (Professors, ECN)
- C. Stolz (Director of Research, CNRS)
- A. Salzman, B. Le (Senior Engineer, ECN)
- B. Shiferaw (phd student)
- F. Cazes, P.-E. Bernard, K. Moreau, G. Rastiello, C. Sarkis, J. Zghal (Former Post-Docs)
- T. Gorris, A.E. Selke, A. Parilla-Gomez (Former Phd students)


CEA IFSTTAR



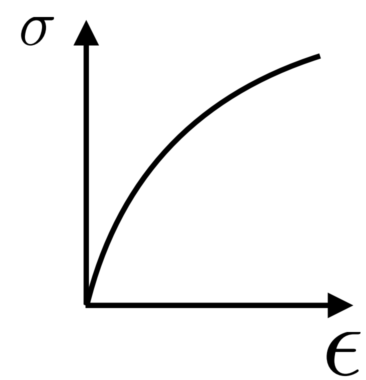
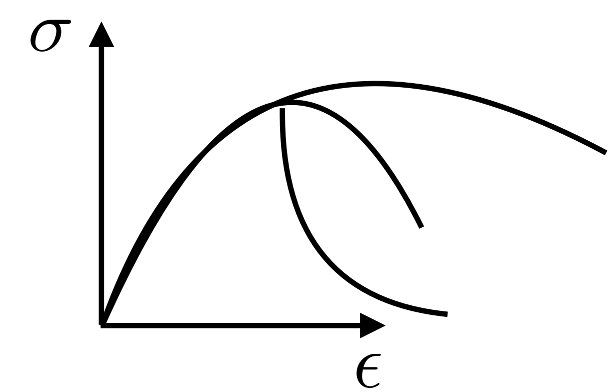

Structural Fracture Simulation Rules of the Game

- Given a structure, a material and a loading, answer the following type of questions:

- Will a crack appear? If yes for what load? Is it fatal ? Where is it going ? How much energy does it take away? (Carpiuc bench for instance, L.Poncelet et al.).

- The (material) model is supposed to be identified on a set of specimen/structural experiments and then used to predict reality in a wide range of different loadings/ geometries.

Classification of approaches for fracture simulation

	Non softening bulk	Softening Bulk
		
No Displacement Discontinuity	No fracture! (or just local criteria)	Gradient/Integral/ Phase-Field
Displacement Discontinuity	LEFM (et corr.plas.) CZM	TLS G/I/PF with added crack

Need add. infos on crack direction, branching?

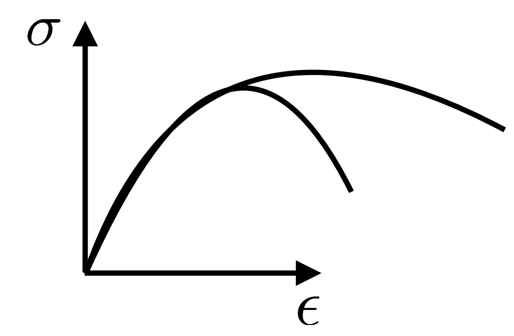
No discontinuity, softening bulk approaches

- **Integral approach**: the damage evolution is governed by a driving force which is non-local i.e. it is the average of the local driving force over some region: (Bazant, Belytschko, Chang 1984, Pijaudier-Cabot and Bazant 1987).
- **Higher order, kinematically based, gradient approach** involving higher order gradients of the deformation: (Aifantis 1984, Triantafillydis and Aifantis 1986, Schreyer and Chen 1986) or additional rotational degrees of freedom (Mühlhaus and Vardoulakis 1987).
- **Higher order, damage based, gradient models**: the gradient of the damage is a variable as well as the damage itself. This leads to a second order operator acting on the damage: (Fremond and Nedjar 1996, Pijaudier-Cabot and Burlion 1996, Peerlings, de Borst et al 1996, Lorentz et Andrieux 1999, Nguyen and Andrieux 2005).
- **Generalized continua, micro-morphic approach** Forest (et al.) 2006
- **Variational approach of fracture:** (Francfort and Marigo 1998, Bourdin, Francfort and Marigo 2000, Bourdin, Francfort and Marigo 2008)
- **Phase-field approach** emanating from the physics community: (Karma, Kessler and Levine 2001, Hakim and Karma 2005) and more recently revisited by (Miehe, Welschinger, Hofacker, 2010).
- **Peridynamics** Silling 2000
- **Comparison papers :** Peerlings, Geers et al. 2001, Lorentz et Andrieux 2003

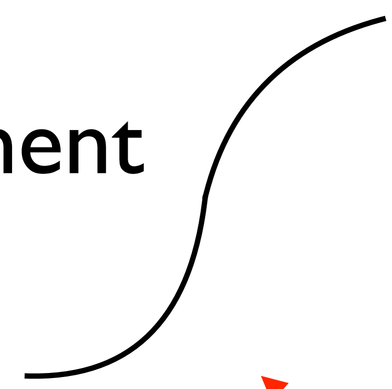
Global regularization and no specific concern for discontinuity

We concentrate on softening bulk to guide the crack

Softening Bulk



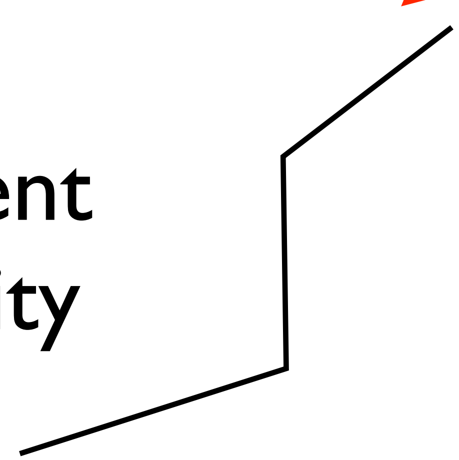
No Displacement Discontinuity



Gradient/Integral/Phase-Field

Choice ?

Displacement Discontinuity



TLS   G/I/PF with added crack

Discontinuity or no discontinuity? Numerical point of view

- No discontinuity requires very small elements to match the high displacement gradients
- Discontinuity allows mesh coarsening away from moving tips
- Discontinuity limits element distorsion with large strains
- Discontinuity is more complicate than no discontinuity but X-FEM is available and remeshing techniques have made a lot of progress over the past decade.
- Discontinuity handling can a priori be tedious with complex crack topologies (we will fix this).

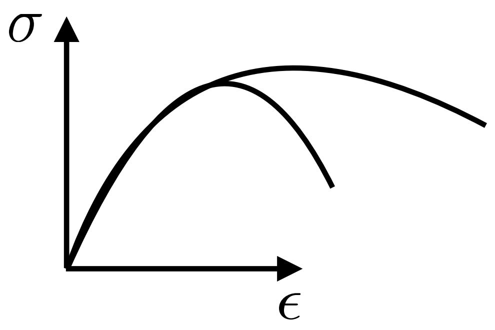
Discontinuity or no discontinuity? Theory point of view

- Discontinuity gives a direct access to crack opening (useful for contact, friction, hydraulic fracture, fragmentation, cutting, blanking,…)

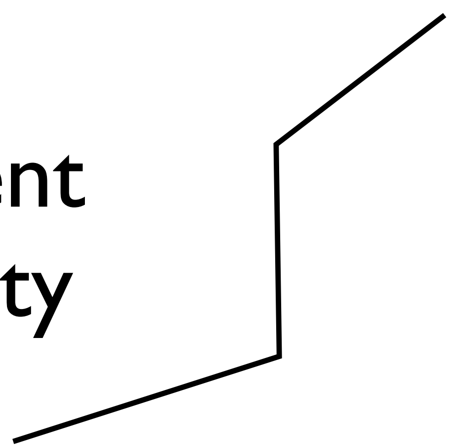
- Discontinuity does not require Gamma Convergence, ie, no need to prove that the formulation mimics a crack opening because there is a crack opening in the formulation

We concentrate on softening bulk and discontinuity

Softening Bulk

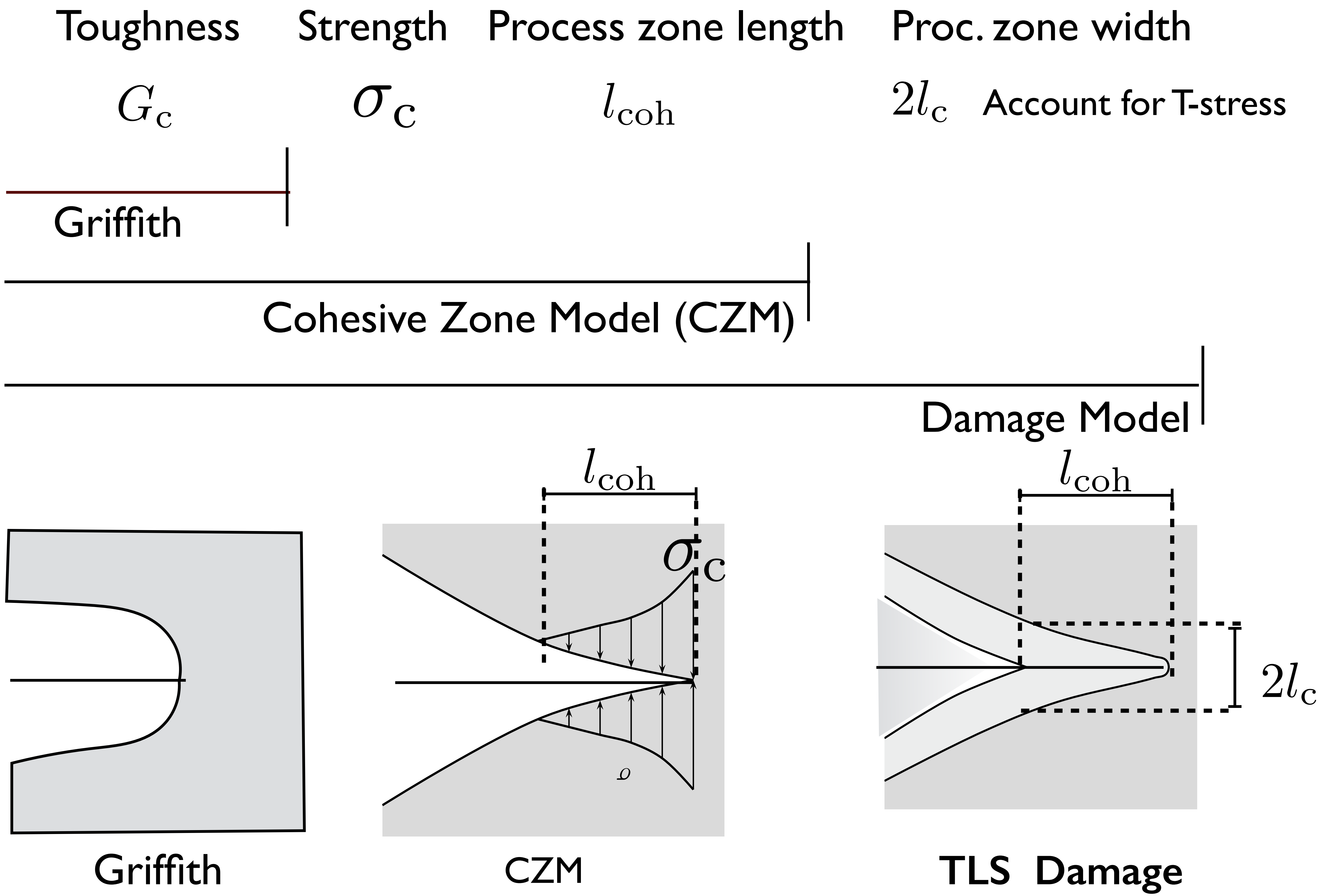


Displacement Discontinuity

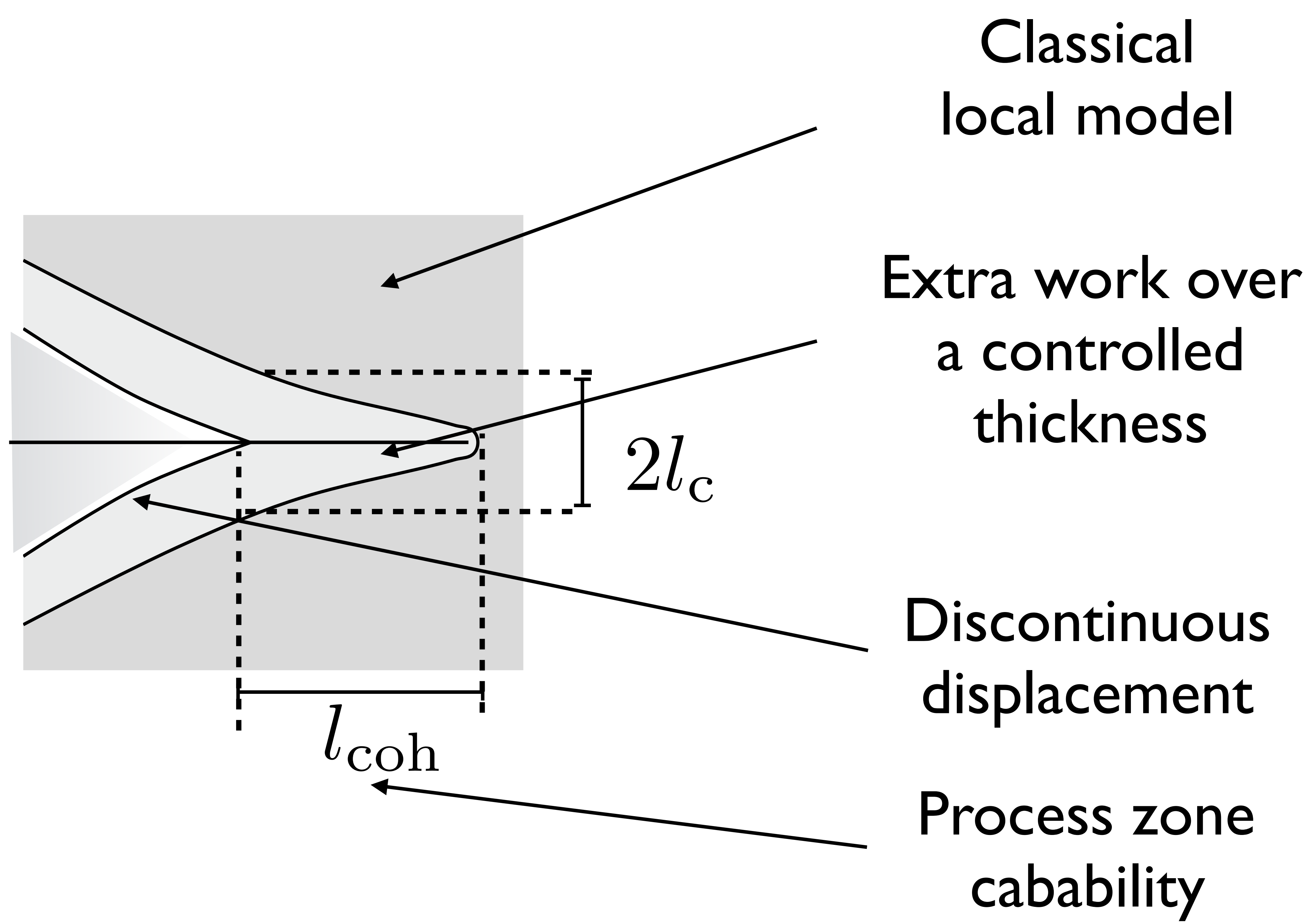


TLS

Quasi-brittle modeling ingredients for a propagating crack

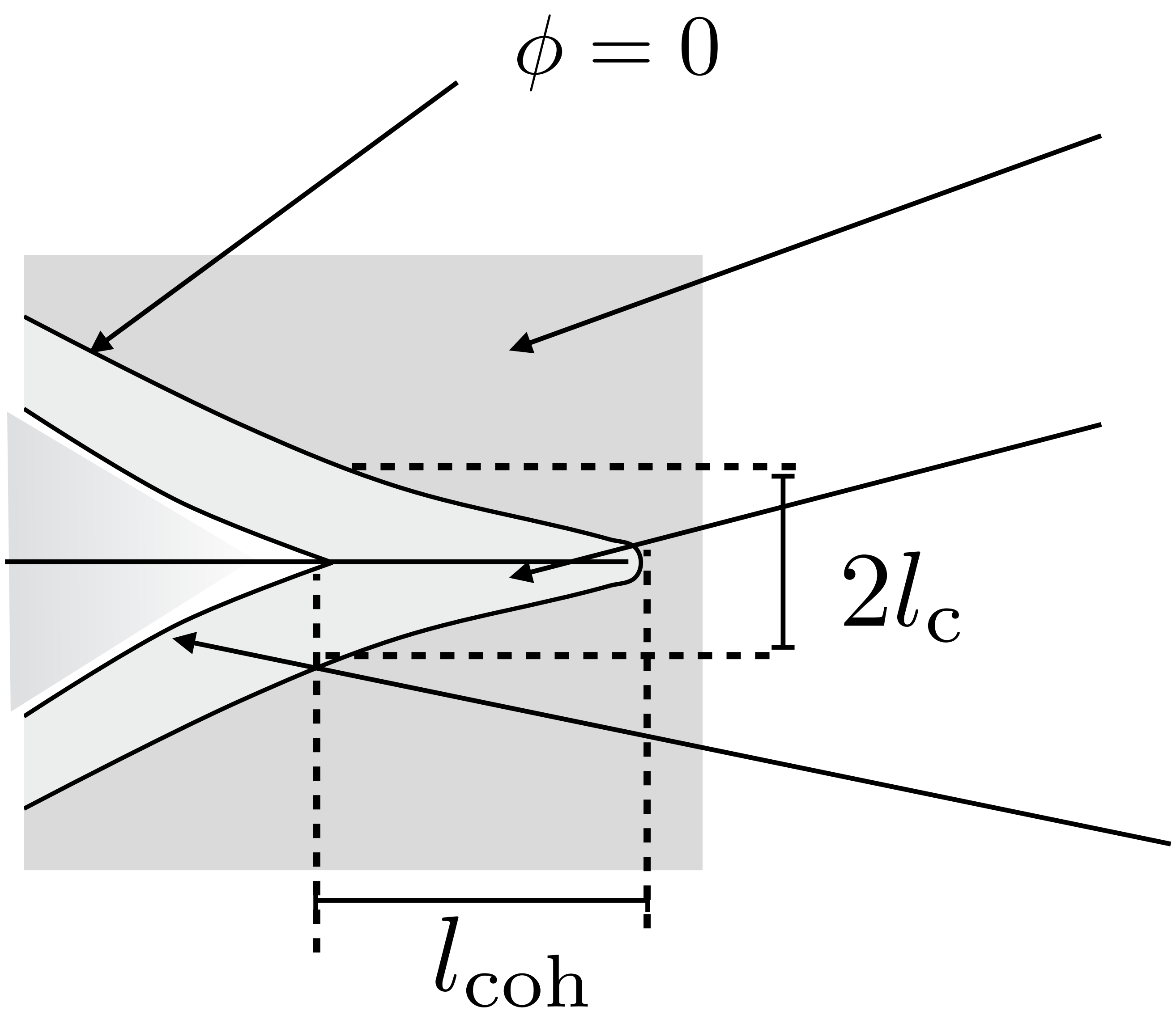


TLS Damage key features



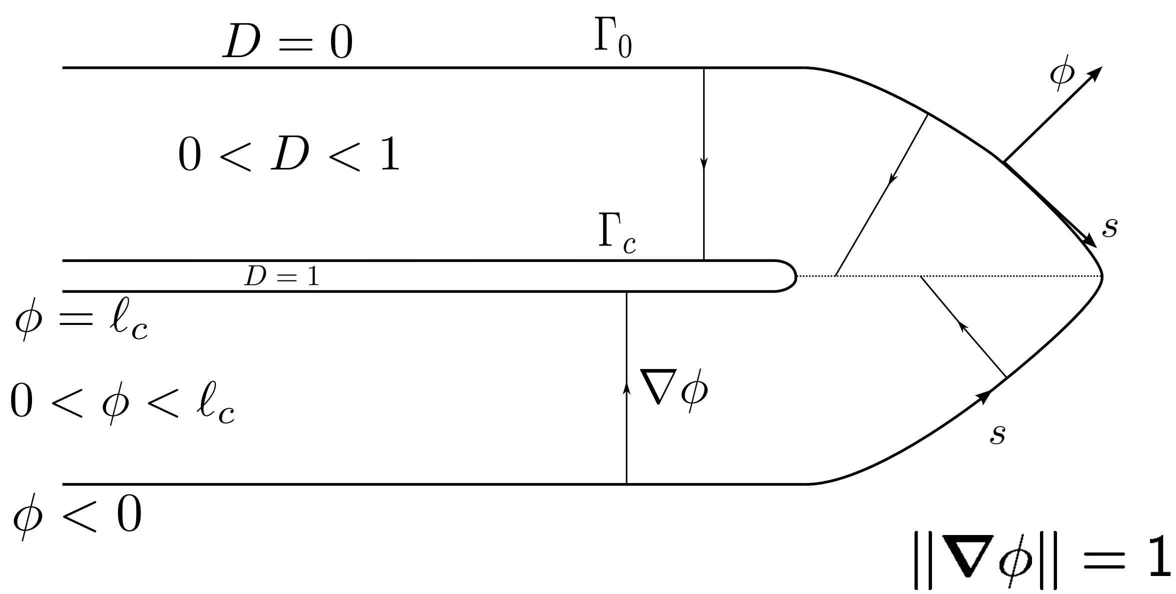
TLS looks like a CZM with some thickness that allows the nose to find its way. This solves the issues of the CZM that lacks directionality for growth at the tip.

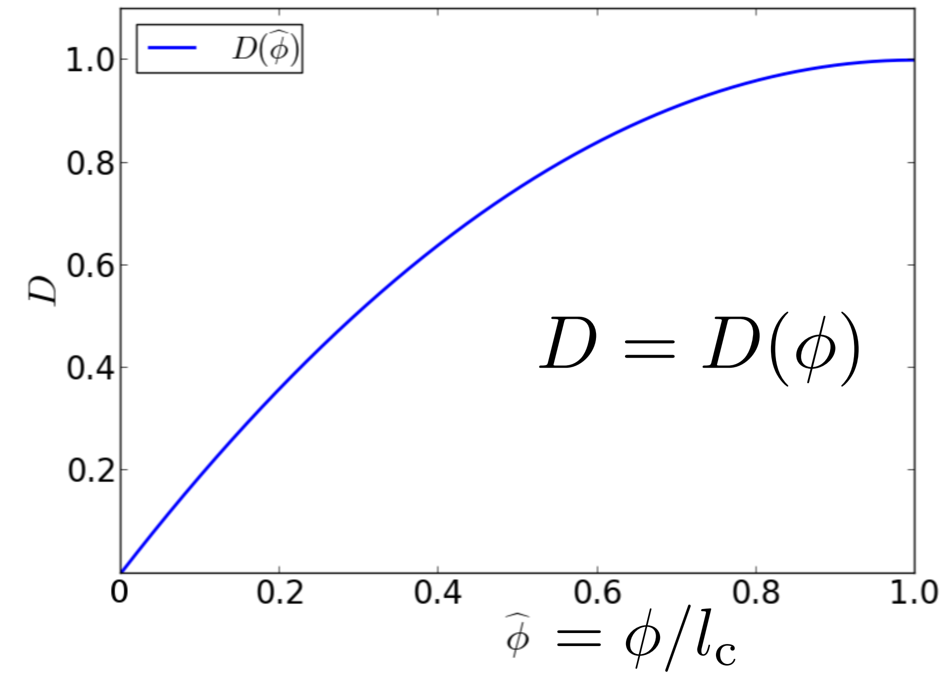
How to make it happen?



Classical local model	$D = 0$
Extra work over a controlled thickness	$D = D(\phi)$
Discontinuous displacement	$D = 1, \phi = l_c$
Process zone cabability	Softening curve

TLS Model Basic Idea :
TLS is a geometrically based damage theory, Zoom on the localizaing zone





1 The damage front is a level set

2 The damage profile is a data of the model

3 Crack faces are thus also given by a level set

TLS equations are thus $D = D(\phi) \quad \|\nabla\phi\| = 1$

or eliminating phi $\|\nabla D\| = \dfrac{g(D)}{l_c}$

TLS theory a priori limits the way damage may evolve

Constitutive Equations (quick summary)

$$\Psi(\epsilon, D) = \frac{1}{2}(1-D)E\epsilon^2$$

and dissymmetric tension-compression

$$\sigma = \frac{\partial \Psi}{\partial \epsilon} = (1-D)E\epsilon, \quad Y = -\frac{\partial \Psi}{\partial D} = \frac{1}{2}E\epsilon^2$$

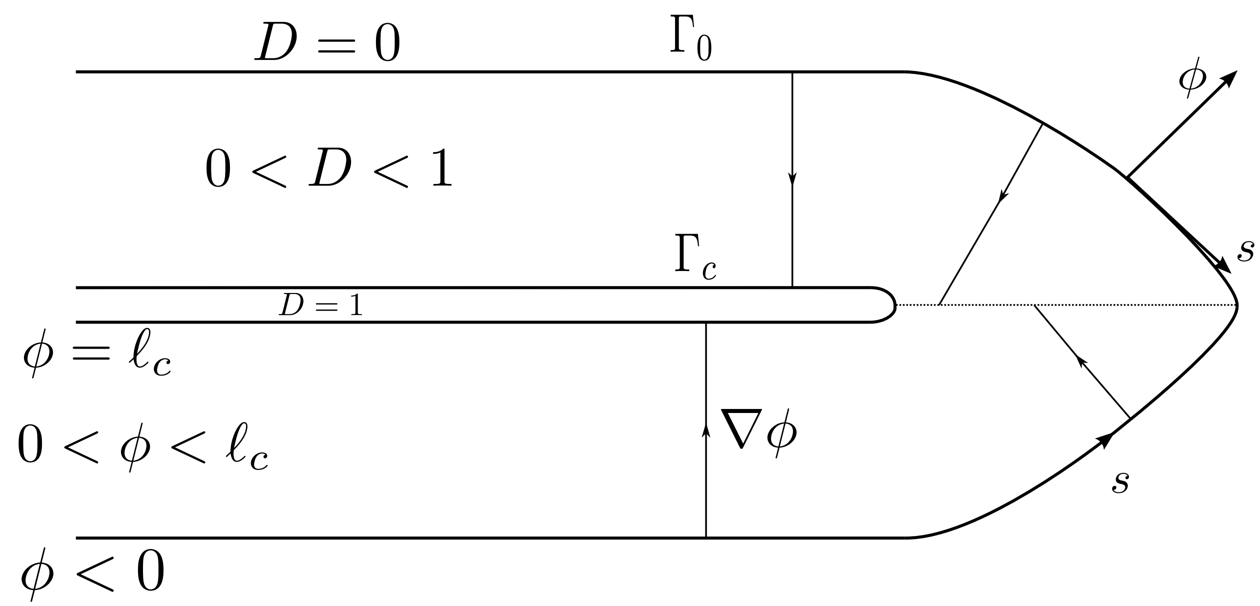
$$\dot{D} \geq 0, \quad Y - Y_c H(D) \leq 0 \quad (Y - Y_c H(D))\dot{D} = 0$$

The last equation is replaced by an averaged one in the localizing zone

$$\overline{\dot{D}} \geq 0, \quad \overline{Y} - Y_c \overline{H} \leq 0 \quad (\overline{Y} - Y_c \overline{H})\overline{\dot{D}} = 0$$

$$Y \to \overline{Y} \to \overline{\dot{D}} \to \dot{D}$$

Mean fields are a consequence on the way damage may evolve



- TLS regularization : replacing local field X by the associated mean field $\overline{X} \in \mathscr{A}$:

$$\int_{\Omega\setminus\Omega_c} \overline{X}\,\overline{X}^* D'(\phi)\mathrm{d}\Omega = \int_{\Omega\setminus\Omega_c} X\overline{X}^* D'(\phi)\mathrm{d}\Omega, \quad \forall \overline{X}^* \in \mathscr{A}$$

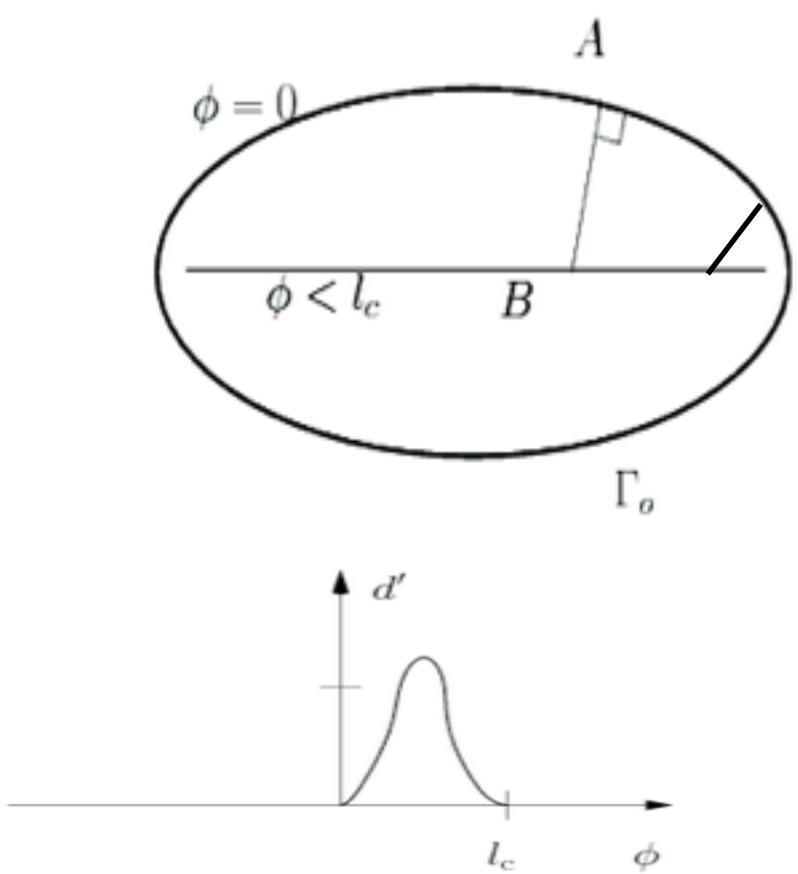
- $\overline{X}$ can be viewed as a weighted mean, computed over segments parallel to the gradient of ϕ:

$$\overline{X}(s) \simeq \frac{\int_0^\ell X(x,s)D' \,\mathrm{d}x}{\int_0^\ell D' \,\mathrm{d}x},$$

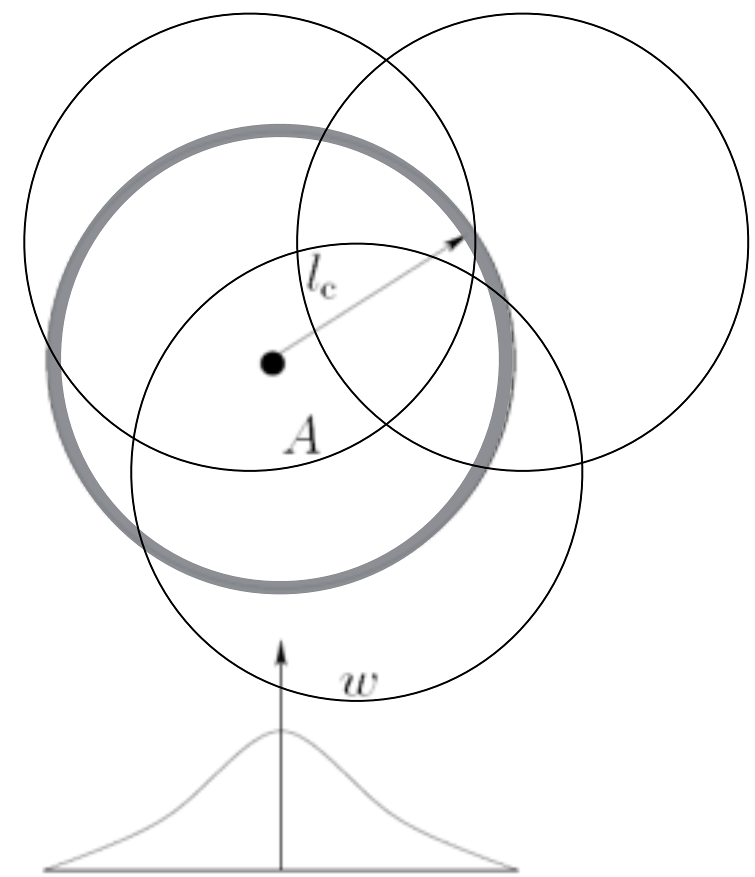
Similarity and difference of TLS with the non-local integral approach

$$\overline{Y}(AB) = \frac{\int_{AB} Y\, d'(\phi)(1-\frac{\phi}{\rho_0})\,\mathrm{d}\phi}{\int_{AB} d'(\phi)(1-\frac{\phi}{\rho_0})\,\mathrm{d}\phi}$$

$$\overline{Y}(A) = \frac{\int_{B_A} Y w\,\mathrm{d}\Omega}{\int_{B_A} w\,\mathrm{d}\Omega}$$



Moes et al. 2011

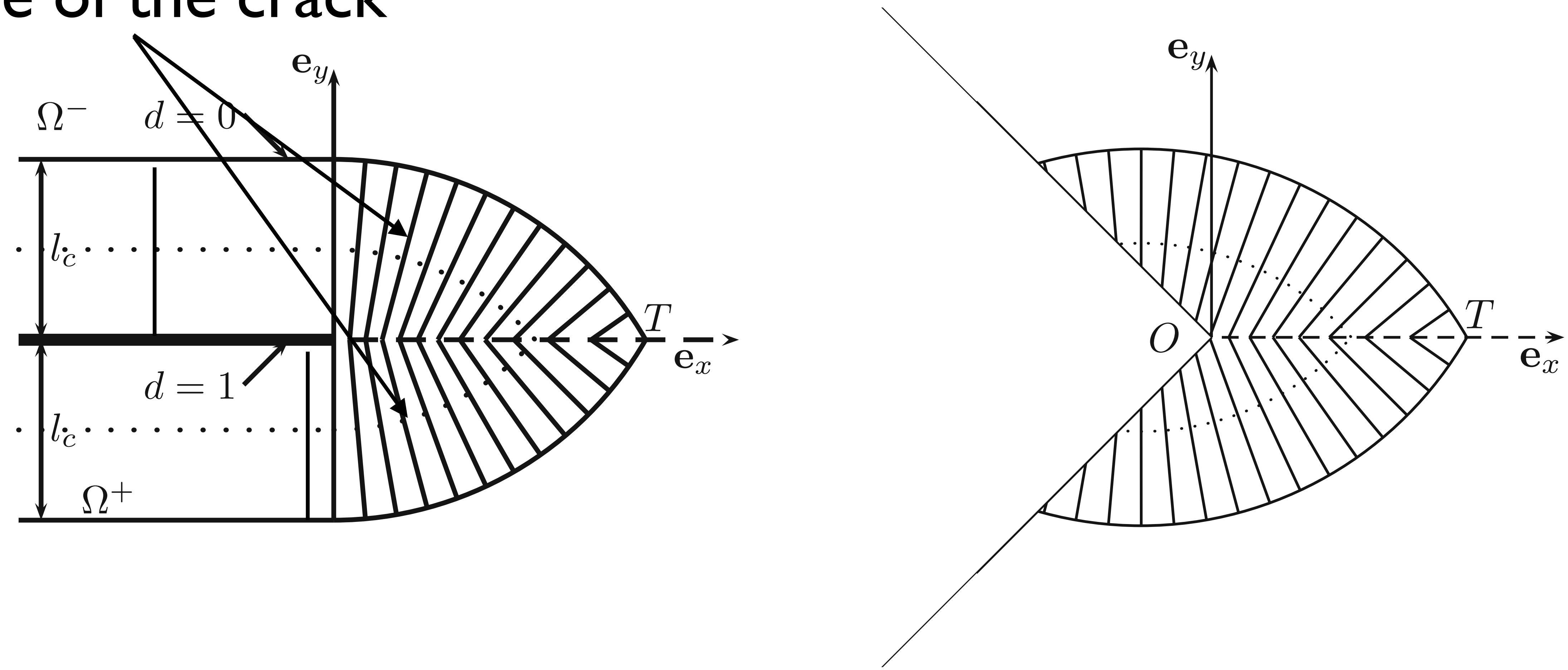


Pijaudier-Cabot, Bazant 1987

In the TLS model, the length over which averaging is performed in non-constant but evolving in time

Segmentation of the localizaing domain to define the non-local driving forces

Indep. segment on each side of the crack



No specific boundary conditions for damage, the damage gradient is not necessarily orthogonal to the boundary (or symmetry plane). Important remark : the segments are not explicitly built for the numerics.

TLS gathers geometrical aspects of fracture and bulk softening of damage

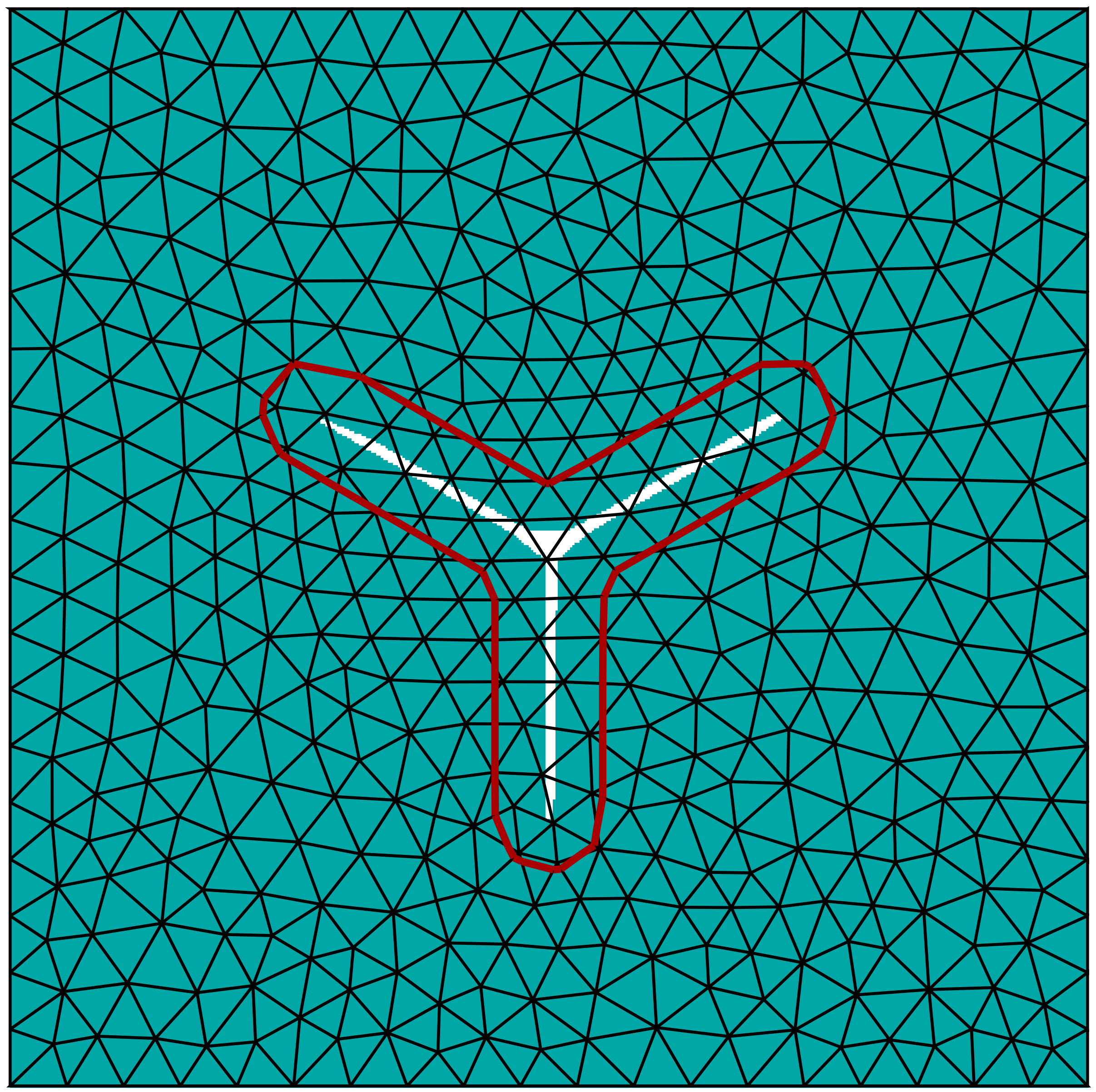
	Fracture	TLS Damage	Damage
Energy	$\int_{\Omega \backslash a} w(u)\ \mathrm{d}\Omega$	$\int_{\Omega} w(u, D(\phi))\ \mathrm{d}\Omega$	$\int_{\Omega} w(u, D)\ \mathrm{d}\Omega$
state equ.	$\sigma = \frac{\partial w}{\partial \epsilon(u)}$	$\sigma = \frac{\partial w}{\partial \epsilon(u)}$	$\sigma = \frac{\partial w}{\partial \epsilon(u)}$
state equ.	$G = -\frac{\partial W}{\partial a}$	$\overline{Y} =< Y >$	$Y = -\frac{\partial w}{\partial D}$
Dissipation	$G\dot{a}$	$\int_{\Omega} \overline{Y}\,\overline{\dot{D}}\ \mathrm{d}\Omega$	$\int_{\Omega} Y\dot{D}\ \mathrm{d}\Omega$
evol. eq.	$\dot{a} = F(G)$	$\overline{\dot{D}} = f(\overline{Y})$	$\dot{D} = f(Y)$

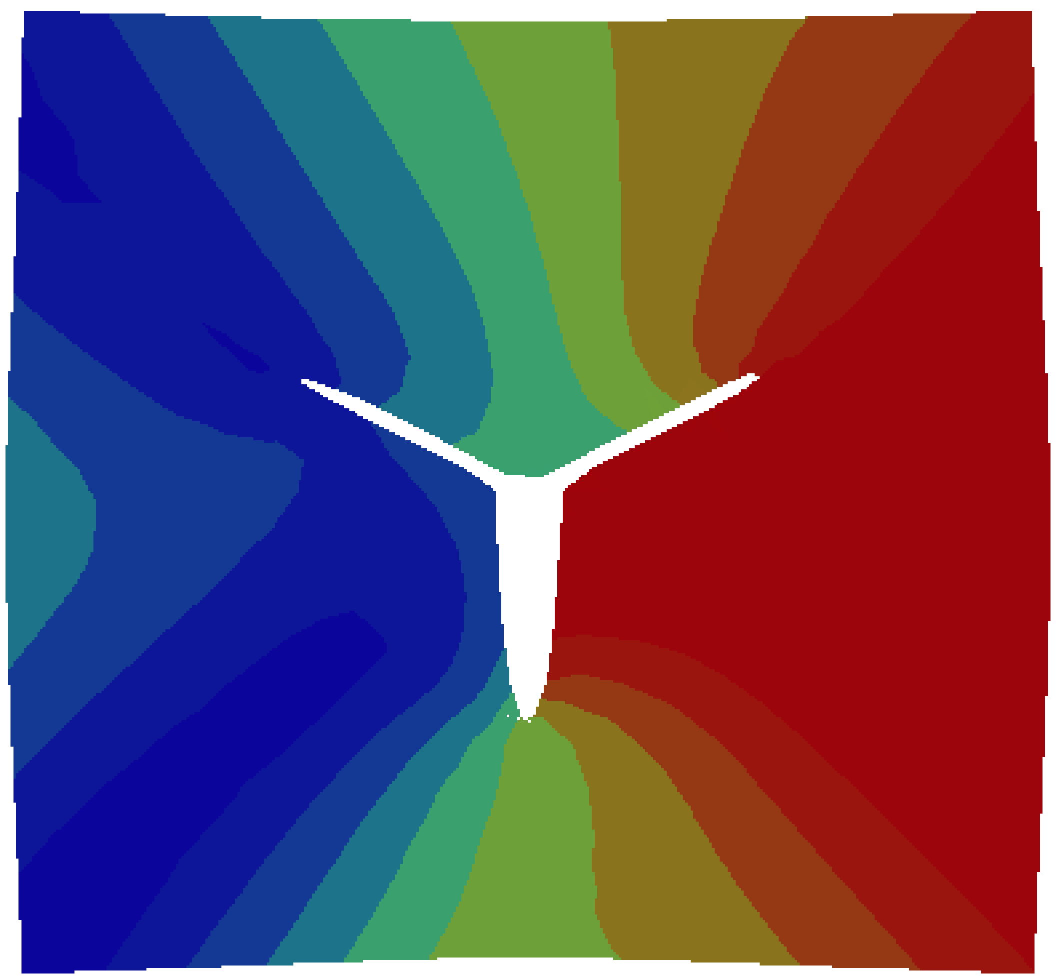
TLS simulation examples

Griffith type Model (short process zone)

Sharp softening

Implementation aspects
X-FEM enrichment to introduce displacement jumps

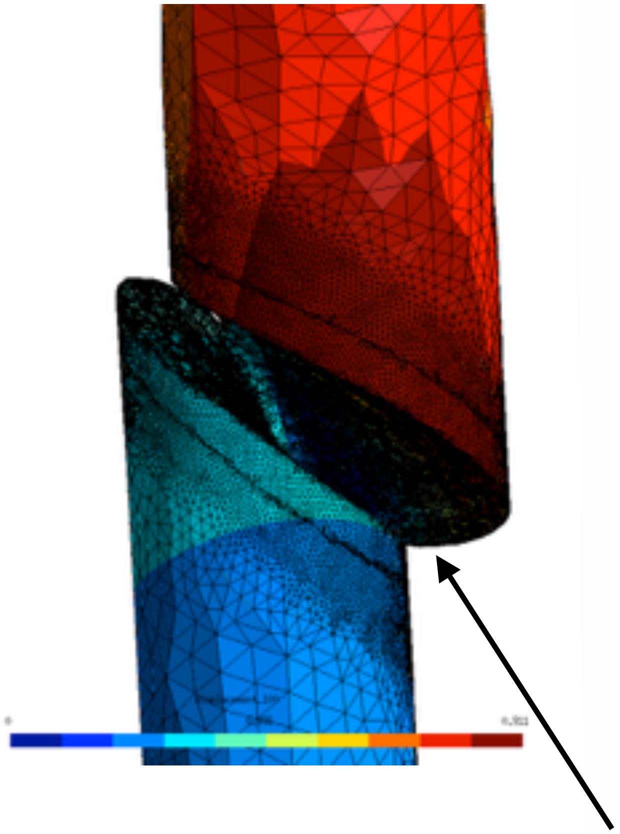


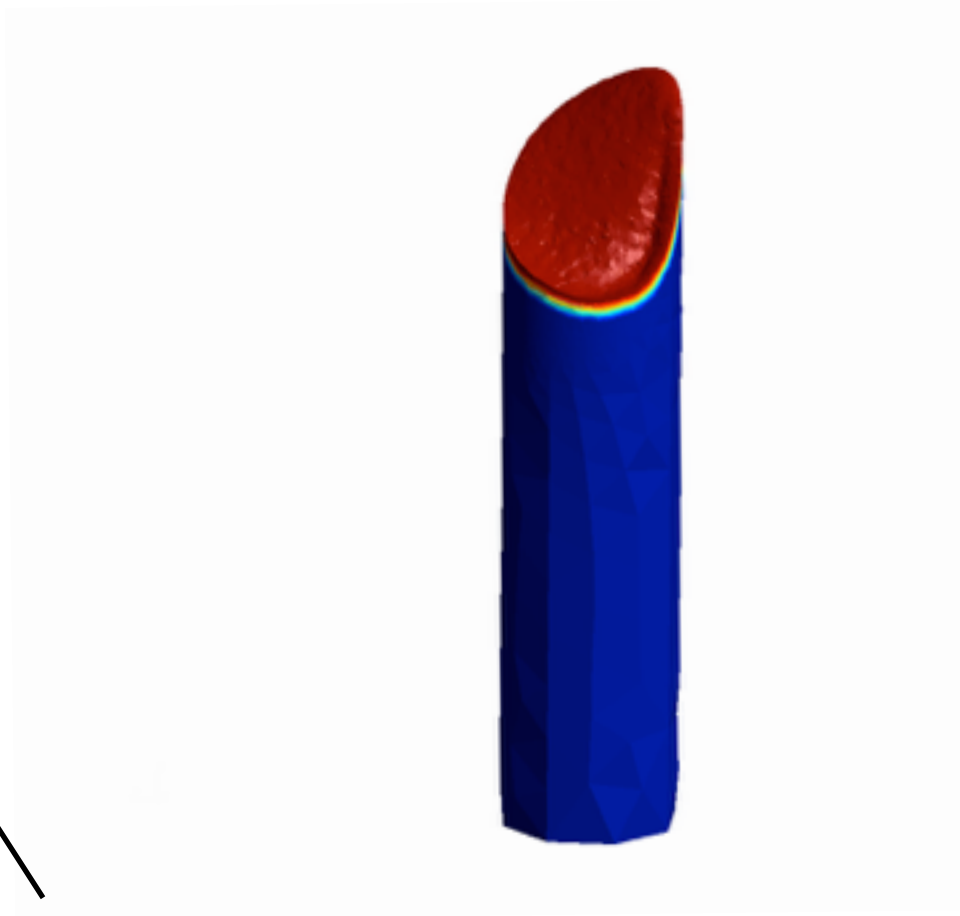


Displacement

X-FEM = Extended finite element method

Simulation examples: 3D chalk case

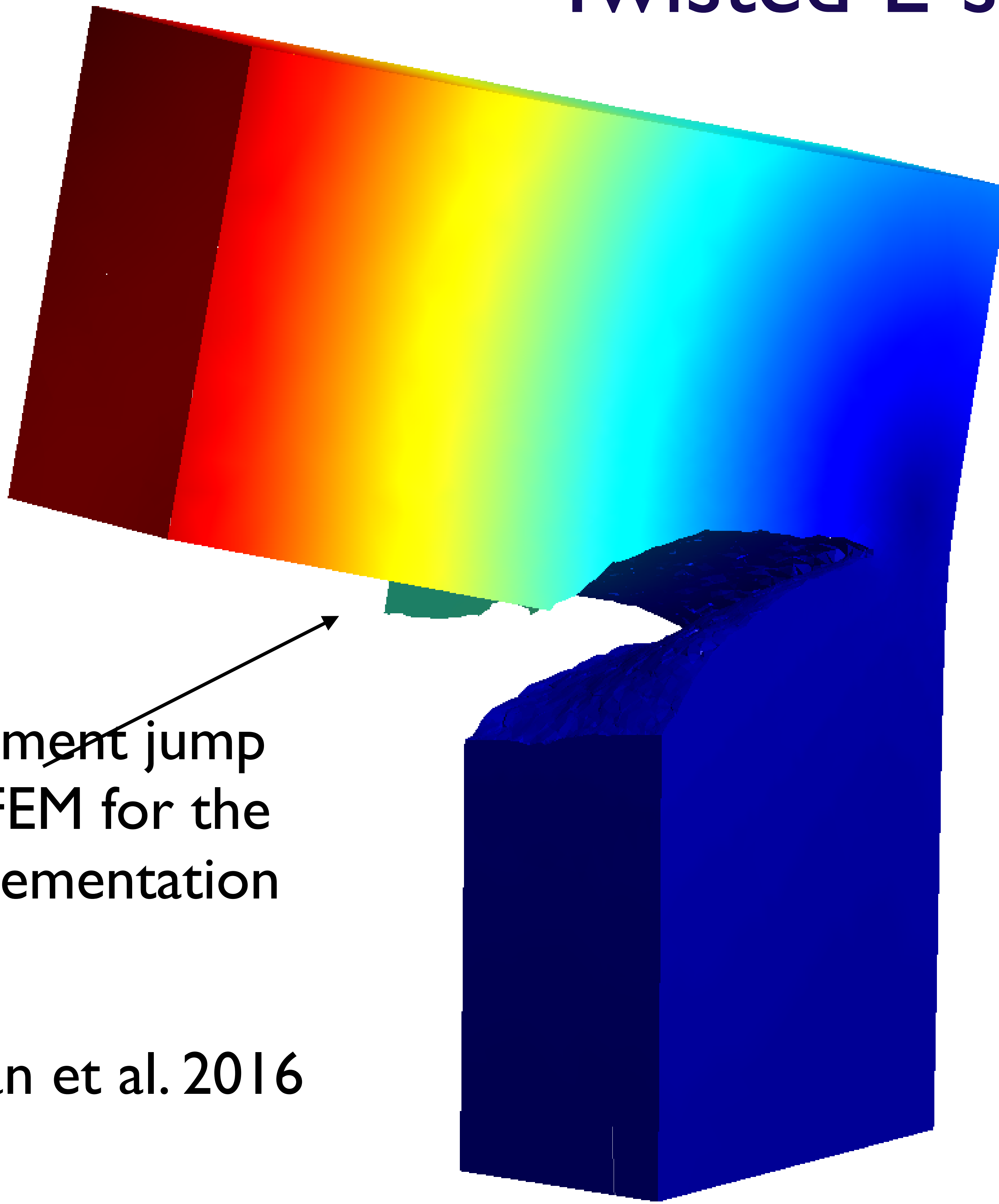




Clear displacement jump

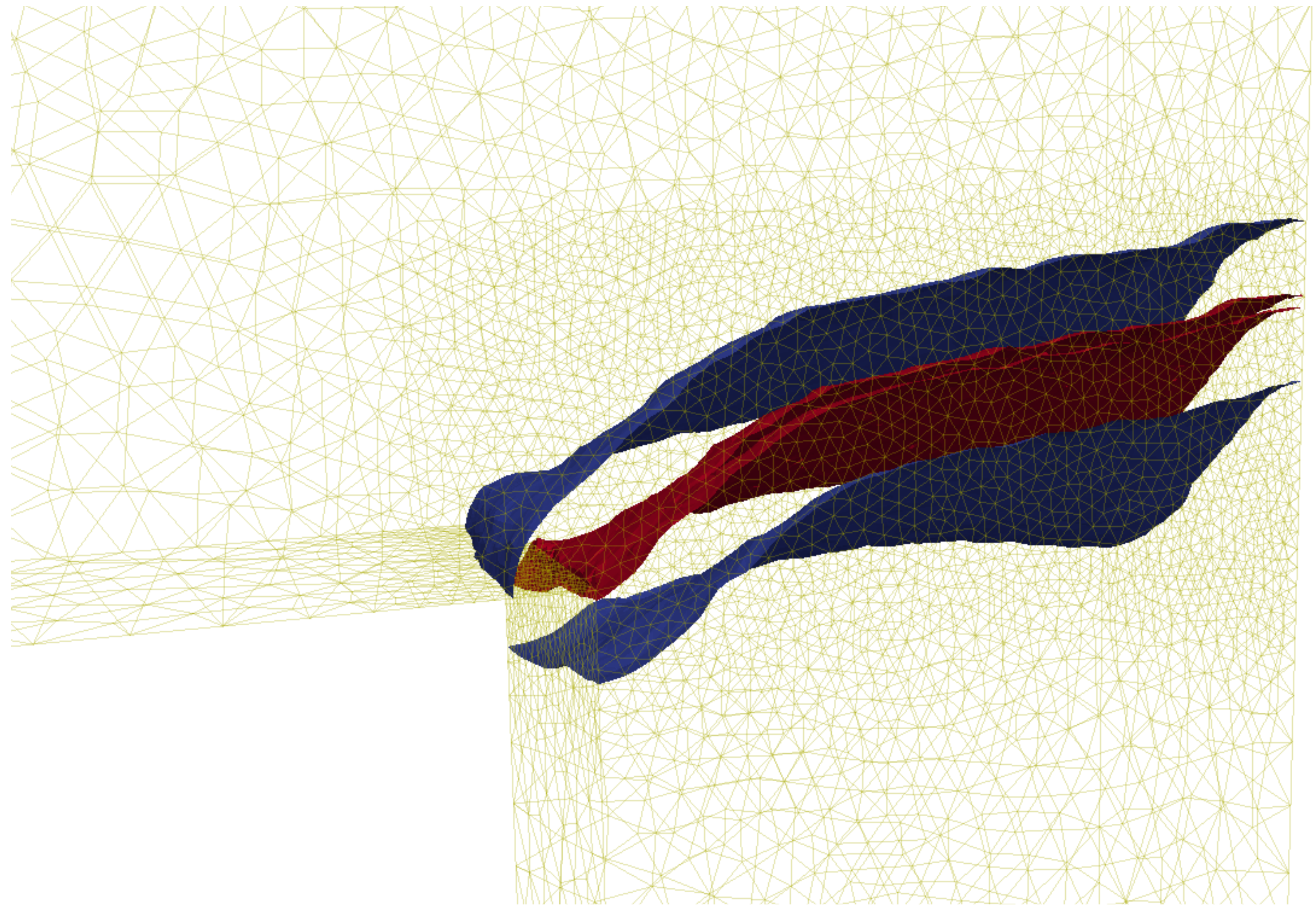
Salzman et al. 2016

Twisted L-shape

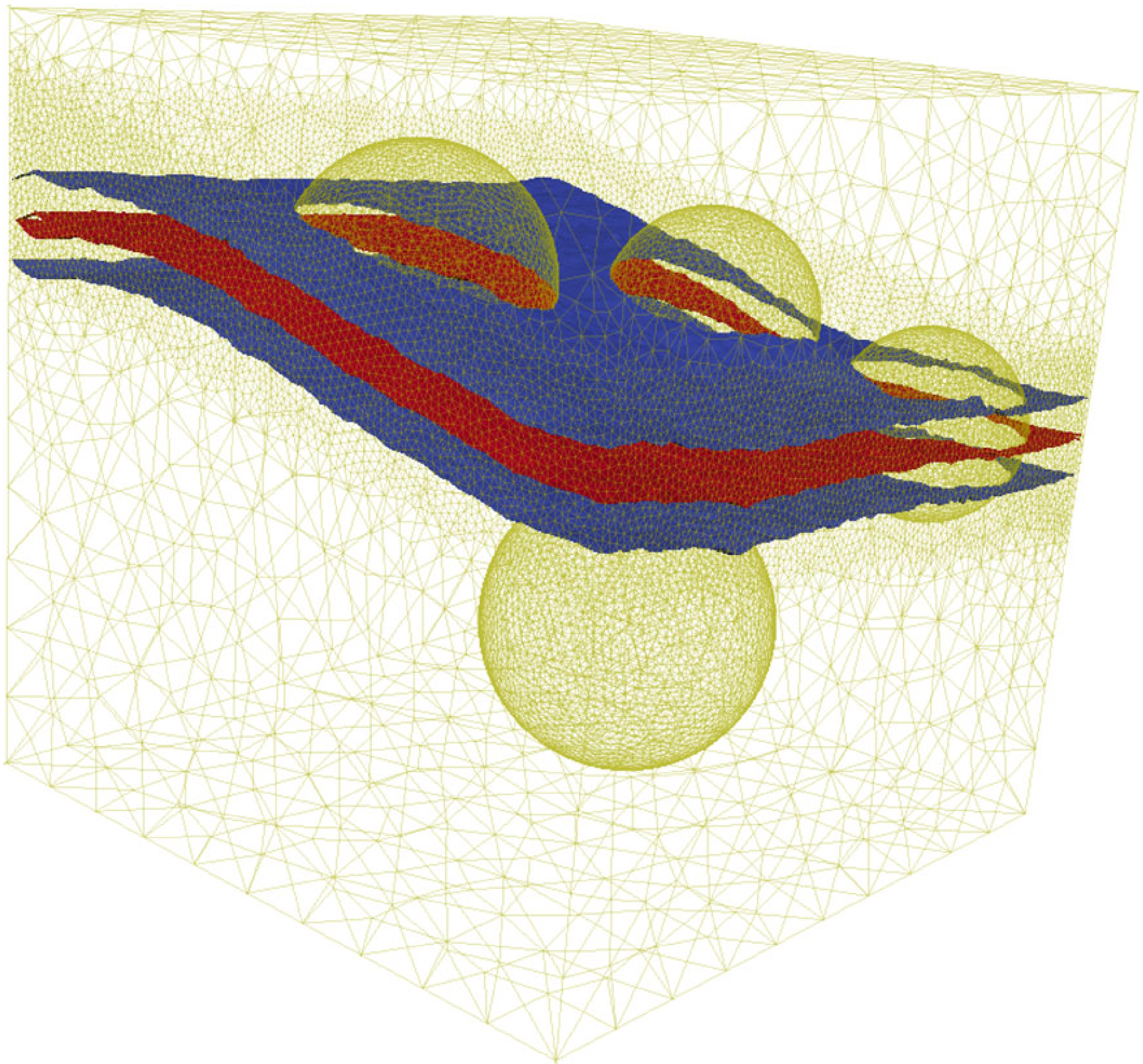


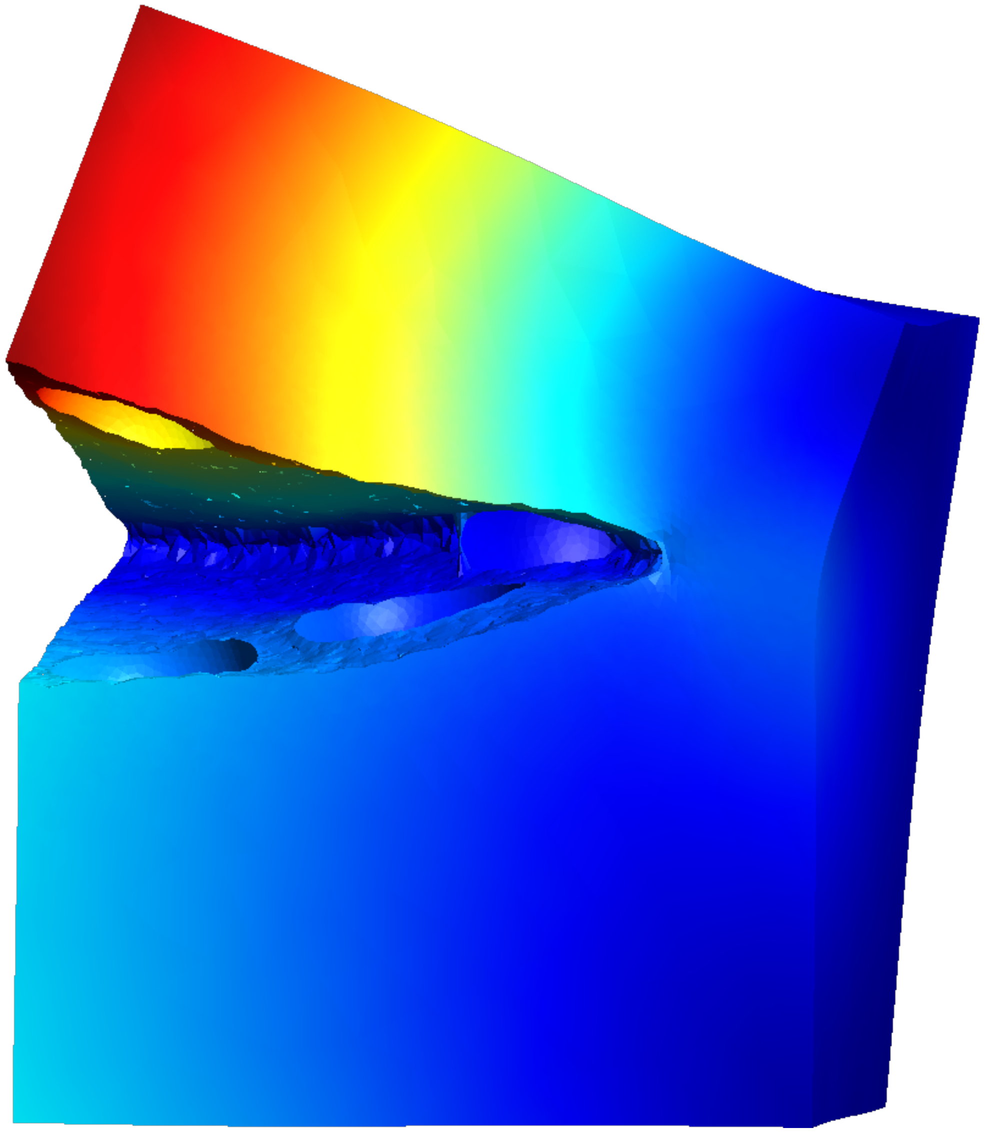
Clear displacement jump
Thanks to X-FEM for the
numerical implementation

Salzman et al. 2016

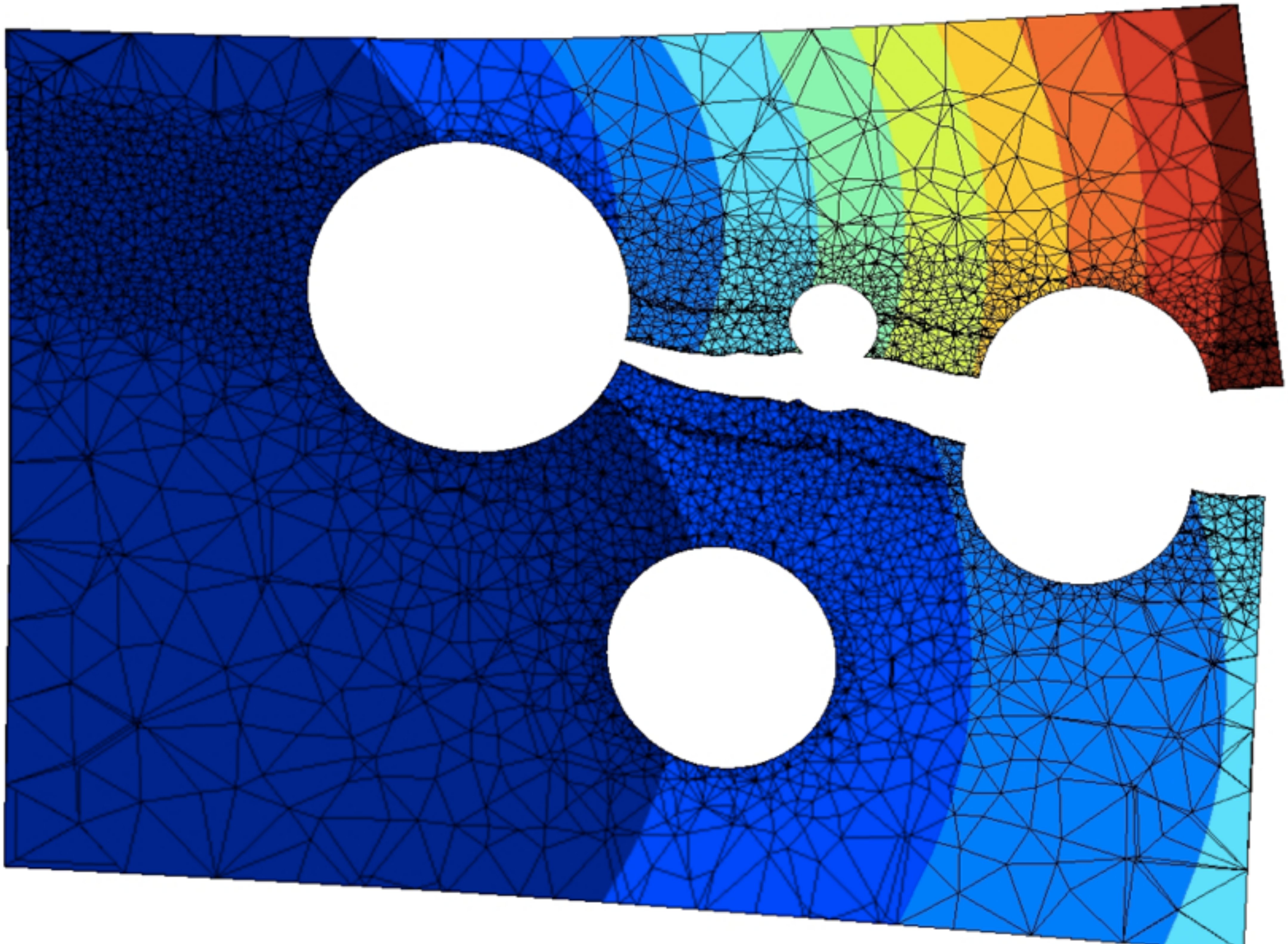


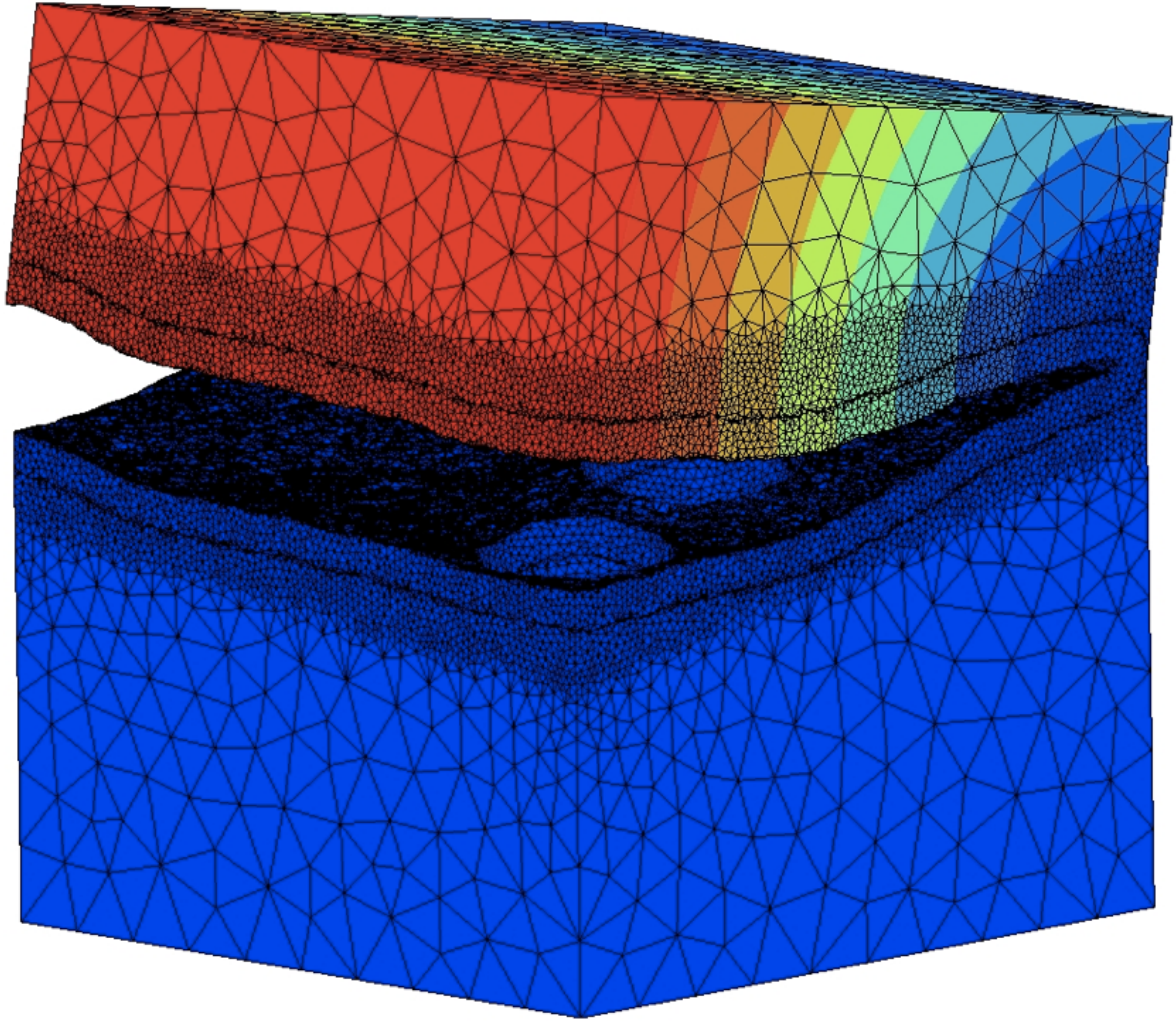
Numerical Cheese





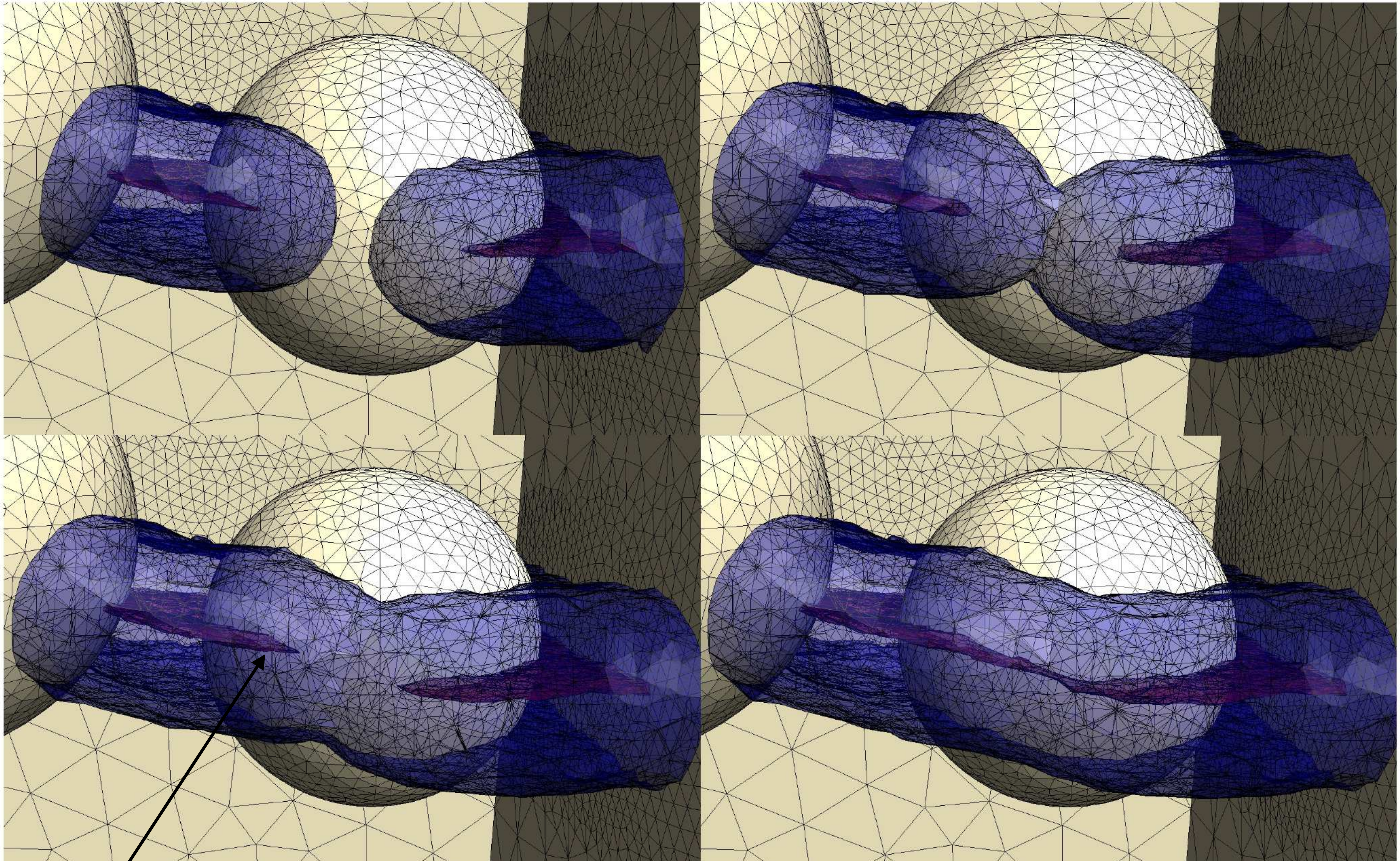
Salzman et al. 2016





displacement 340

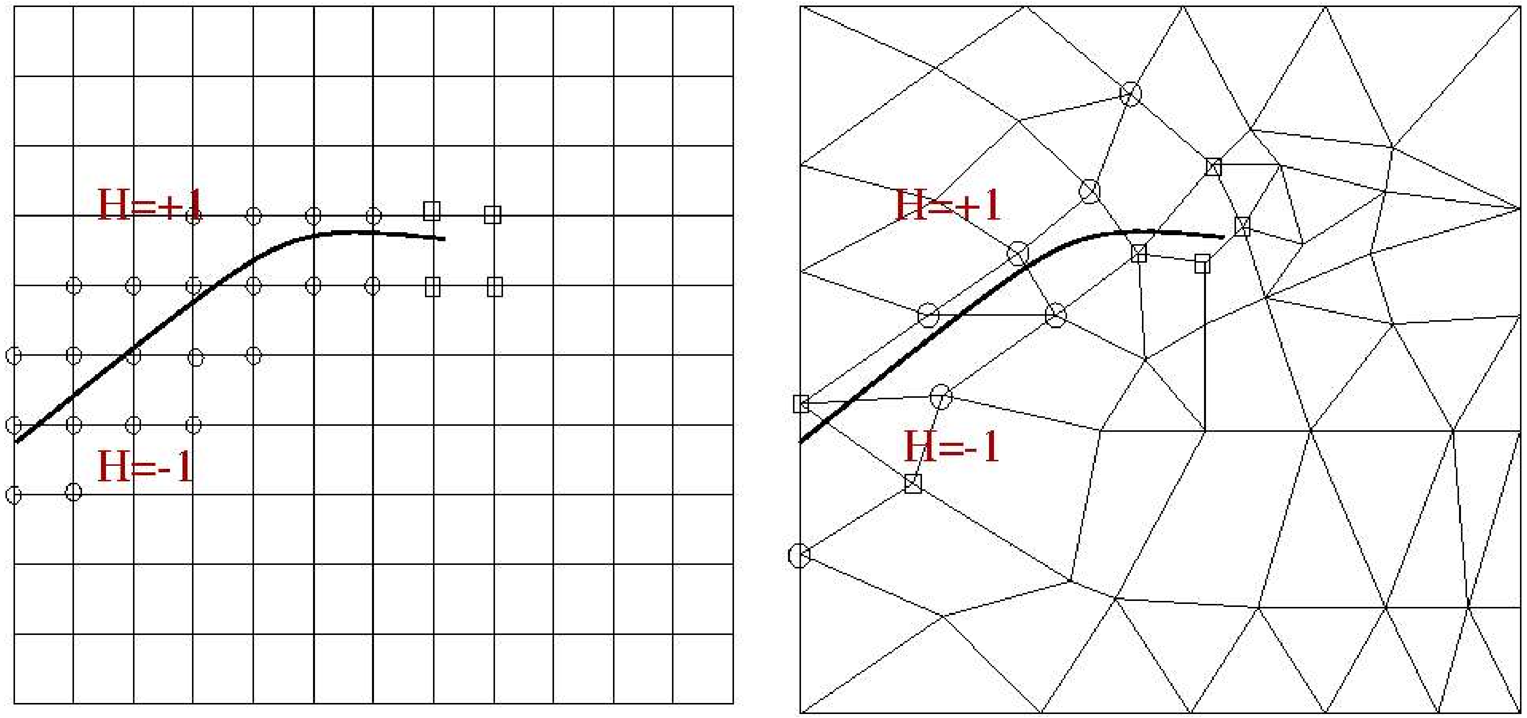
Merging of damage zone, followed by merging of cracks



Non-local computational effort only inside the blue zone

Comparison with LEFM X-FEM simulation of the early 2000’

Modeling cracks with X-FEM



Belytschko & Black 1999

Moës, Dolbow & Belytschko 1999

Sukumar et al. 2000

$$u^h = \underbrace{\sum_{i \in I} u_i \phi_i(\boldsymbol{x})}_{\textit{Classical}} + \underbrace{\sum_{j \in J} b_j \phi_j(\boldsymbol{x}) H(\boldsymbol{x})}_{\textit{Internal Enrichment}} + \underbrace{\sum_{k \in K} \phi_k(\boldsymbol{x}) \left(\sum_{l=1}^{4} c_k^l F_l(\boldsymbol{x}) \right)}_{\textit{Front Enrichment}}$$

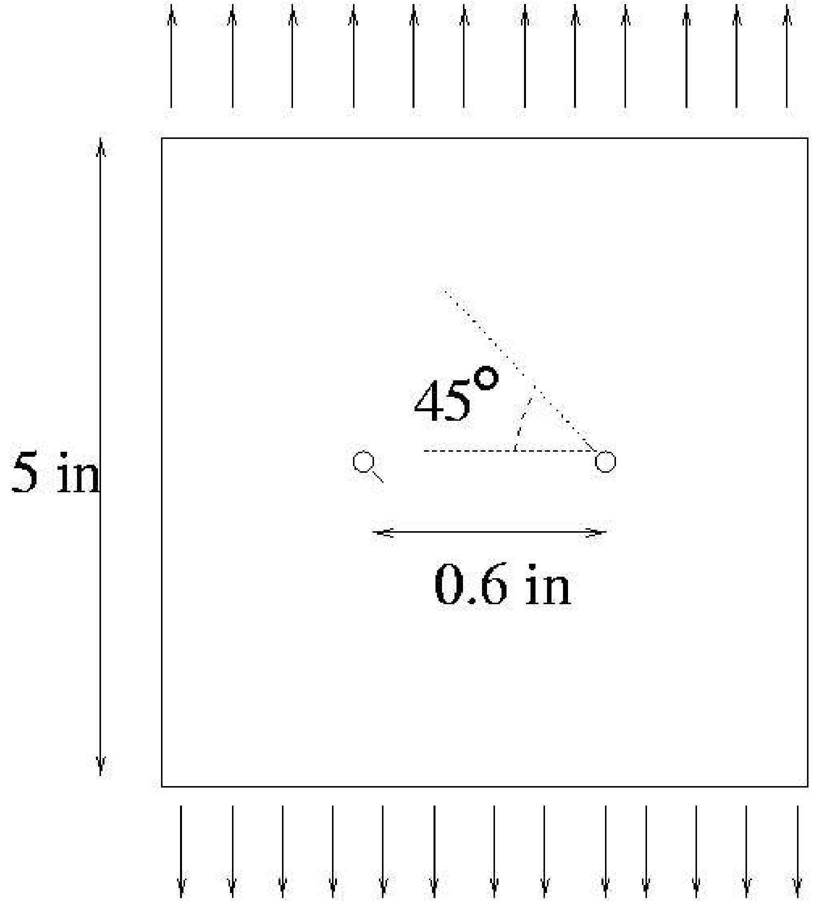
LEFM

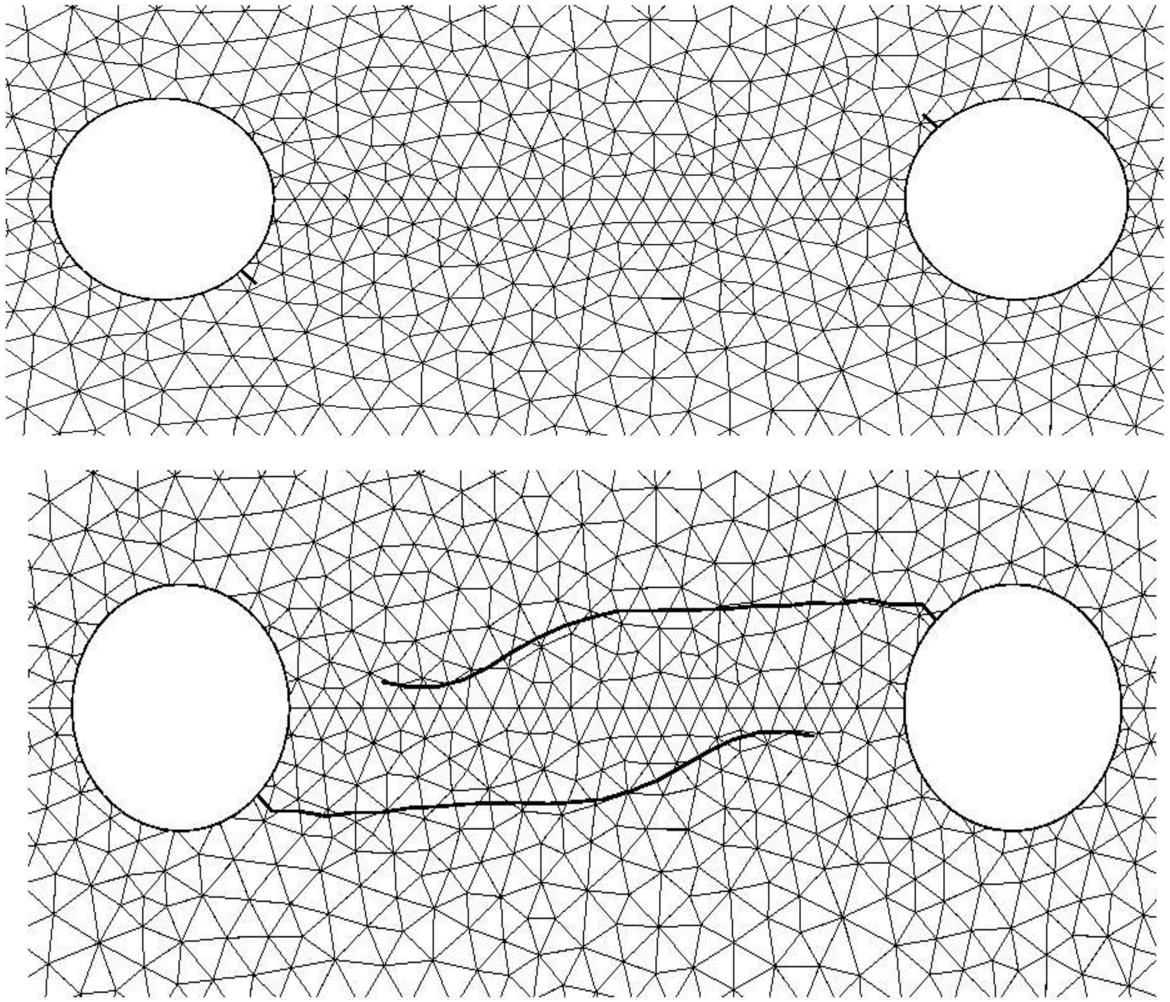
$$\{F_l(\boldsymbol{x})\} = \{ \underbrace{\sqrt{r}\sin(\frac{\theta}{2})}_{\textit{Discontinuous}}, \underbrace{\sqrt{r}\cos(\frac{\theta}{2}), \sqrt{r}\sin(\frac{\theta}{2})\sin(\theta), \sqrt{r}\cos(\frac{\theta}{2})\sin(\theta)}_{\textit{Continuous}} \}$$

Cohesive Zone

$$\{F_l(\boldsymbol{x})\} = \{ r\sin(\frac{\theta}{2}) \text{ or } r^{3/2}\sin(\frac{\theta}{2}) \text{ or } r^2 \sin(\frac{\theta}{2}) \}$$

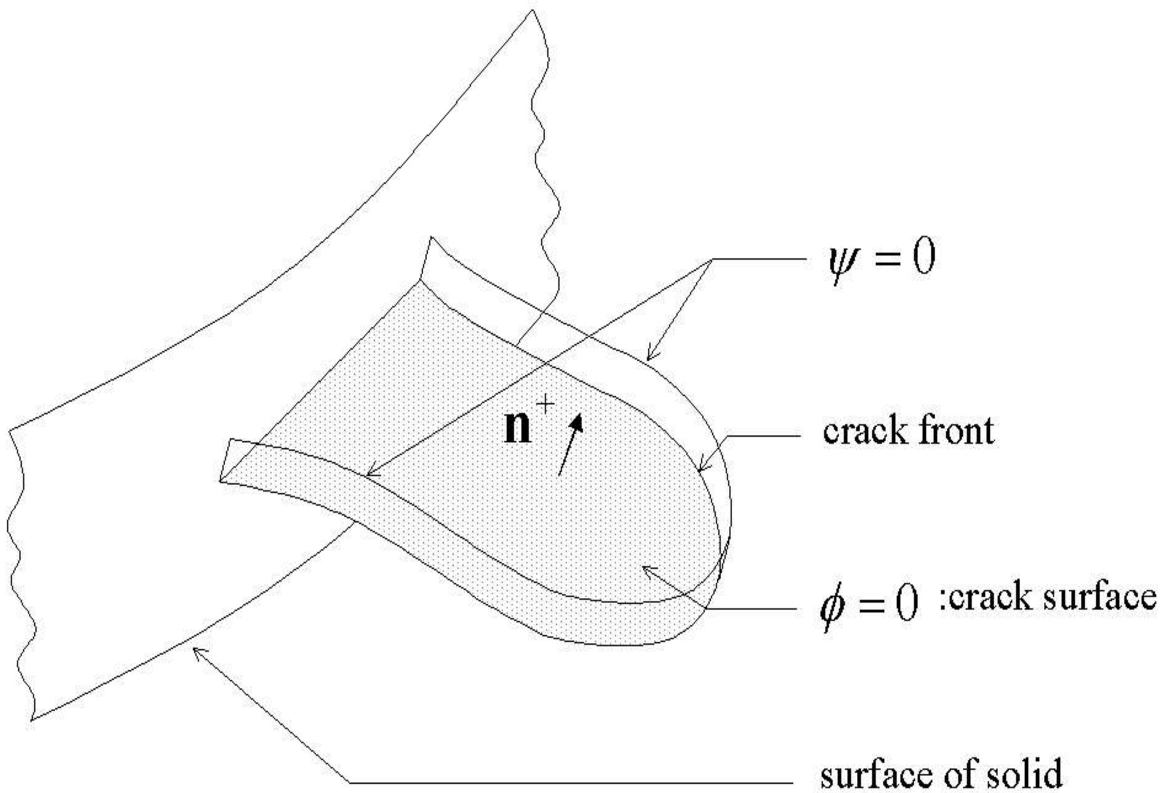
Propagation of two cracks emanating from holes





Belytschko et al. 2000

Level Set Description of the Crack



The level set function are assumed to be orthogonal

$$\nabla \phi \cdot \nabla \psi = 0 \ \forall t$$

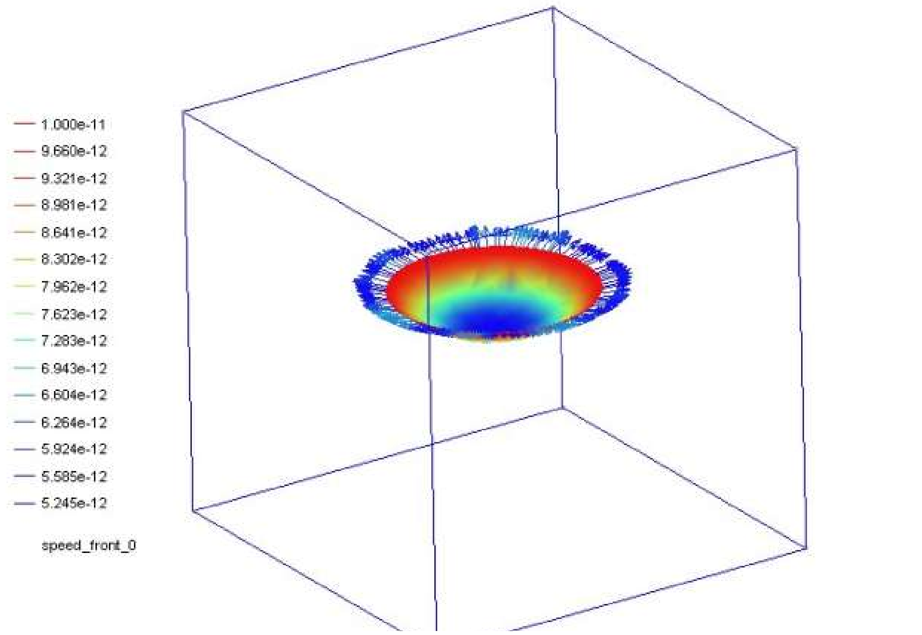
$\phi(\boldsymbol{x}, t), \psi(\boldsymbol{x}, t) < 0$ Defines the crack location

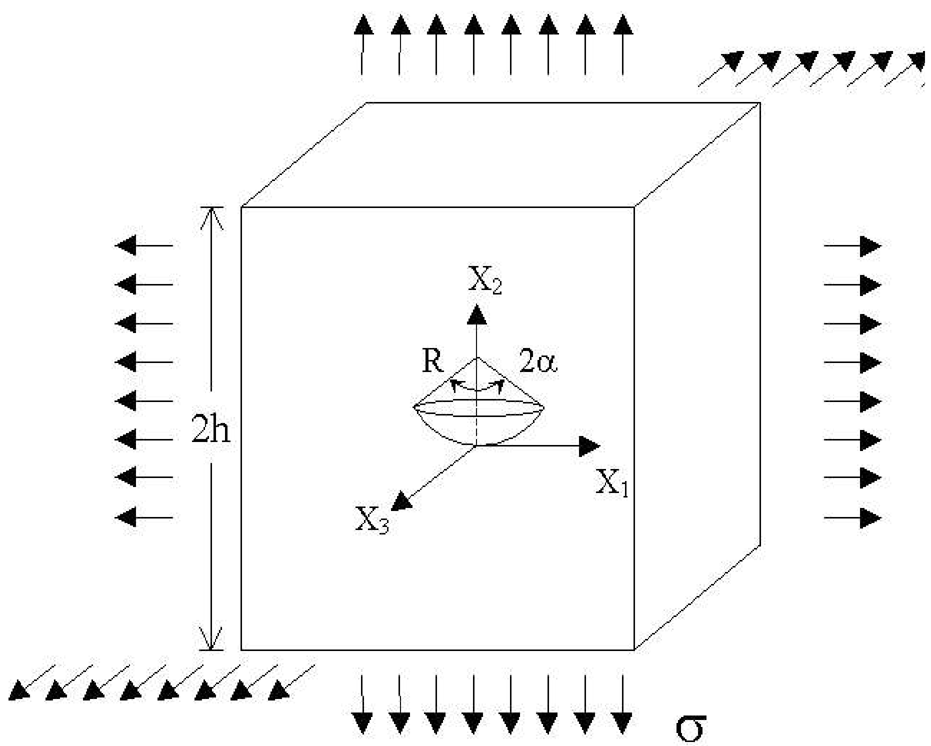
$\phi(\boldsymbol{x}, t), \psi(\boldsymbol{x}, t) = 0$ Gives the crack front

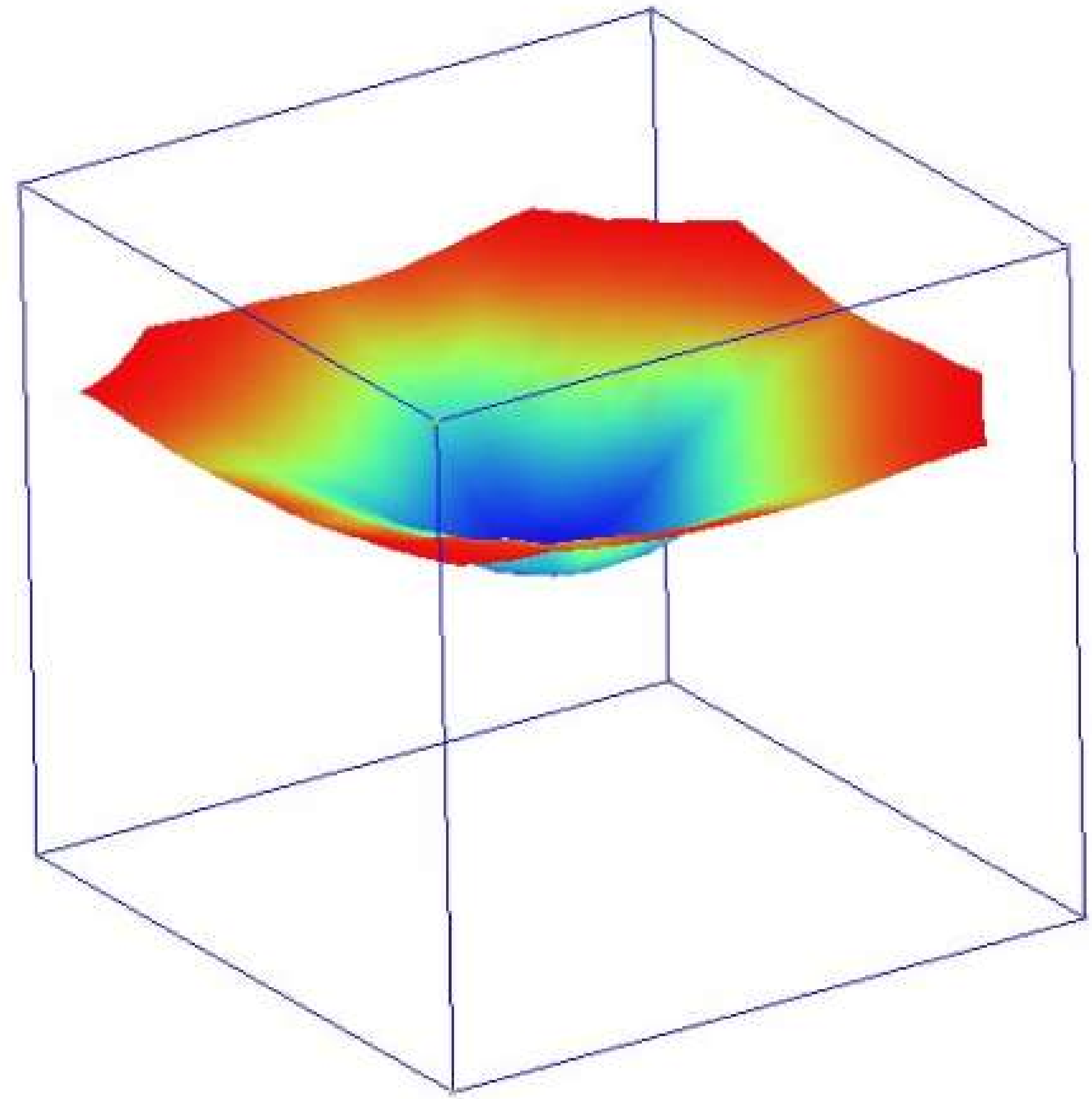
$\psi(\boldsymbol{x}, t) > 0$ does not intersect the crack

Stolarska et al. 2001, Belytschko et al. 2001, Moës et al. 2002

Crack growth : Lens-shaped crack



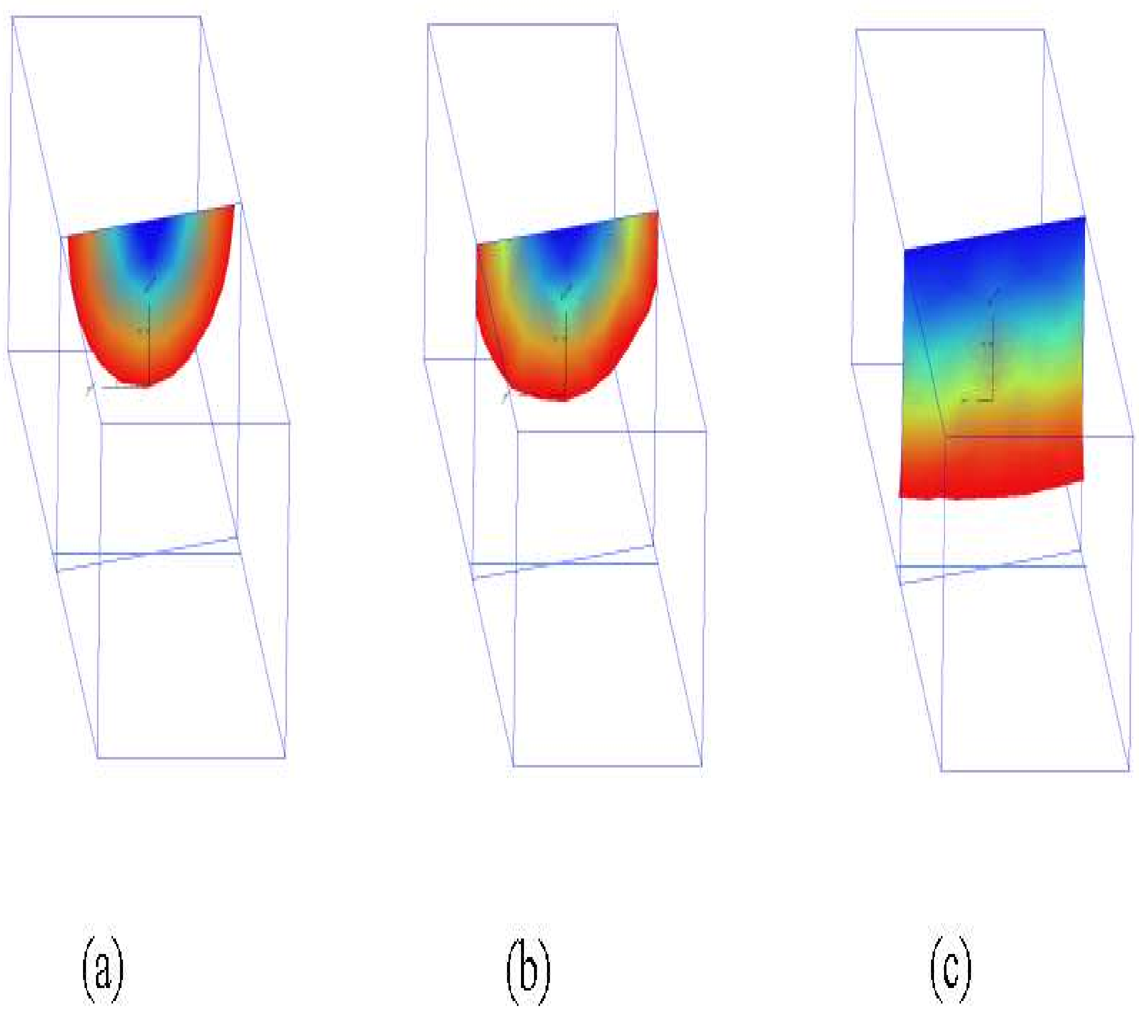


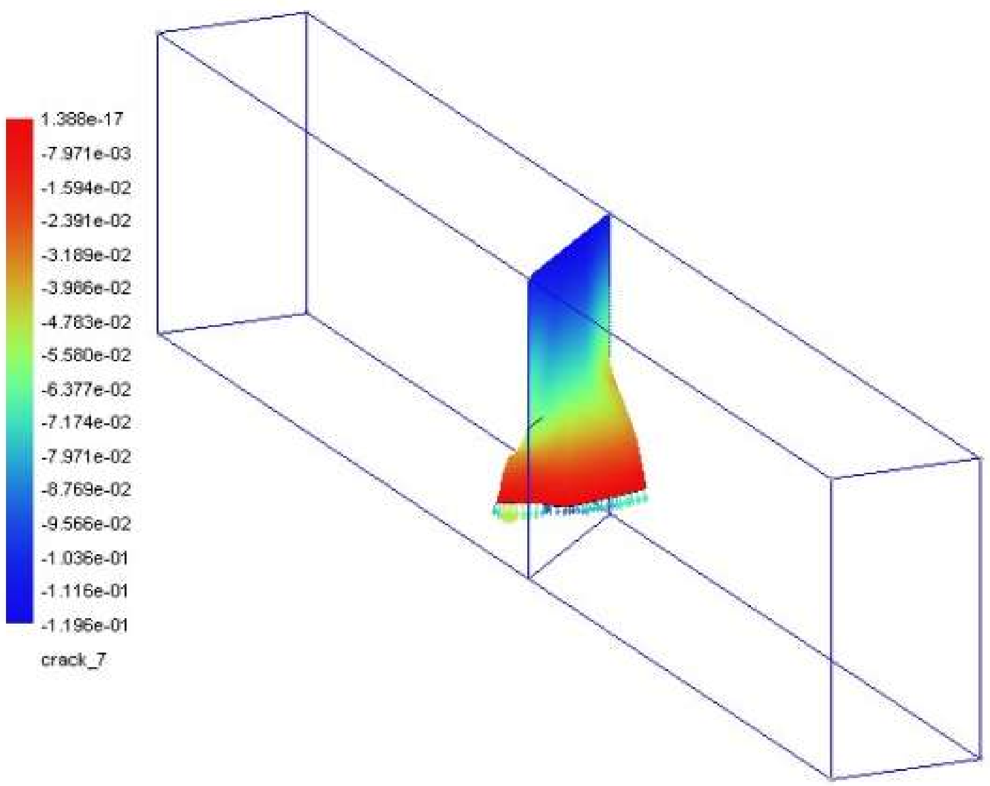


Notice the change in topology of the crack front 1 front, then 4 fronts

Gravouil et al. 2002

Crack growth : Cracked beam in bending





The crack front is rotating as it moves downward to reach mode I

Gravouil et al. 2002

Comments on previous LEFM X-FEM simulations

- Need for an initial crack (no crack initiation)
- Crack growth based on stress intensity factor (not damage based model).
- Two level set fields needed for each crack.
- Crack merging is complex because each independent crack had 2 level set fields.
- X-FEM is now used as a core tool in the TLS implementation. The TLS says where to put the crack.

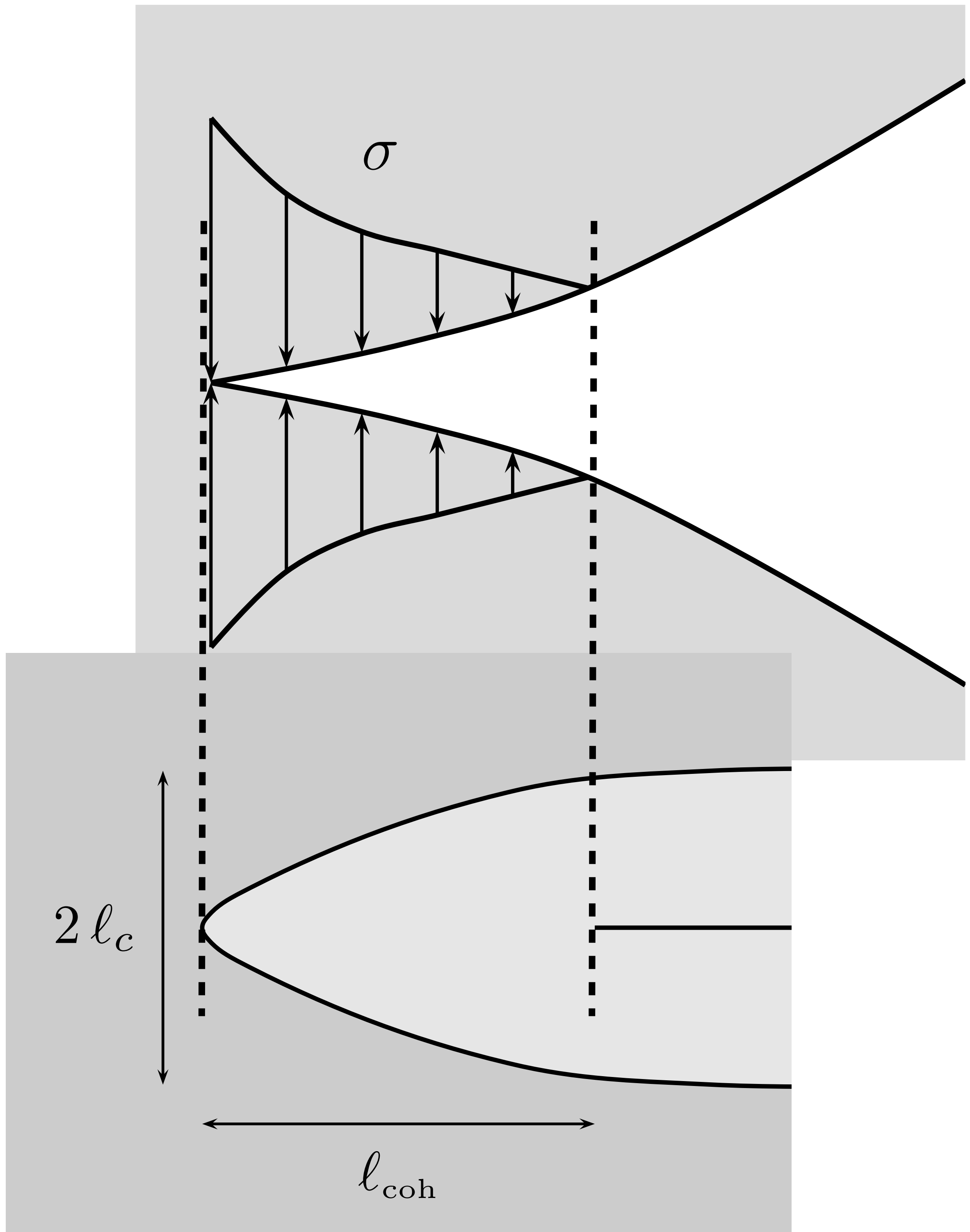
TLS simulation examples

Long process zone and size effect

Geometrical and mechanical similarities

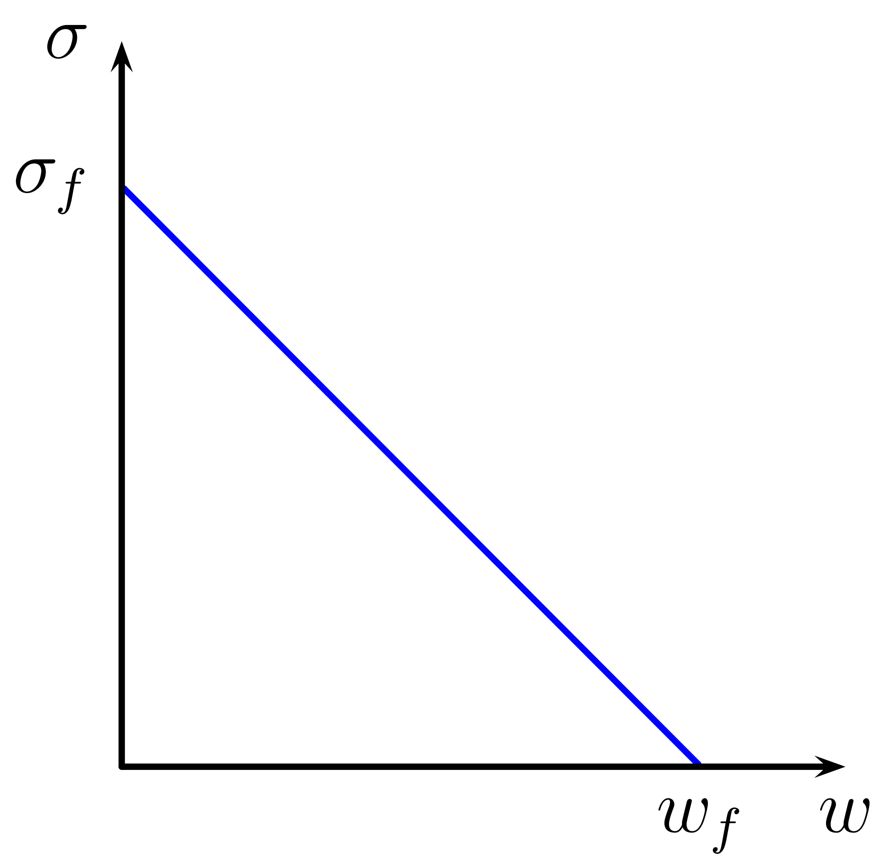
CZM

TLS

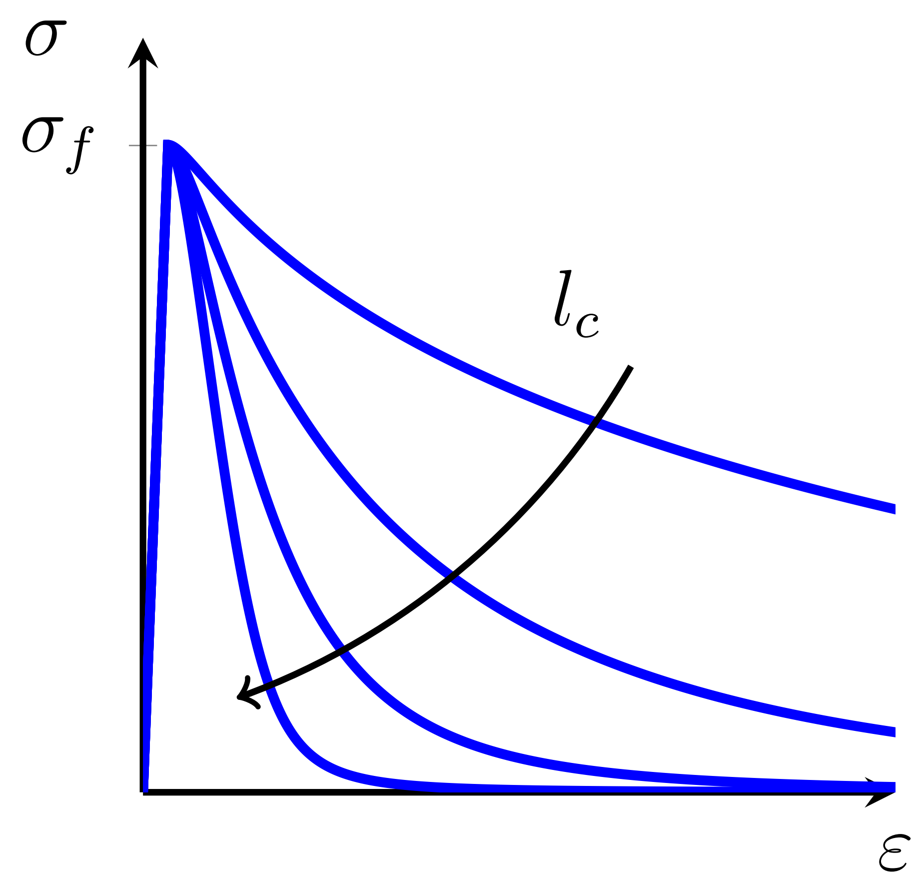


Do we preserve lcoh as lc -> 0 ?
YES

From CZM to TLS



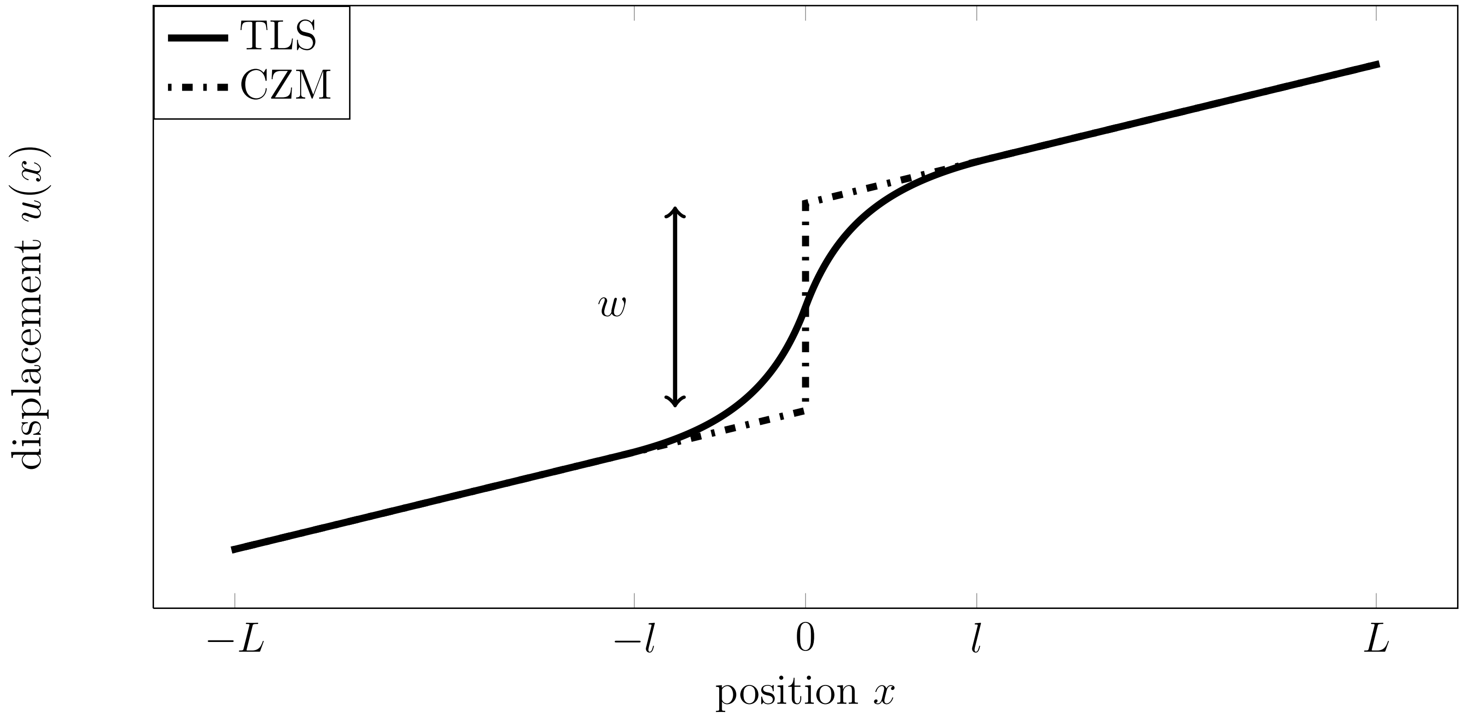
(a) Cohesive linear law.



(b) TLS equivalent local behavior for different ℓ_c values. Increasing values of ℓ_c are indicated by the arrow.

A. Parrilla-Gomez et al. 2015

CZM and TLS 1D equivalence



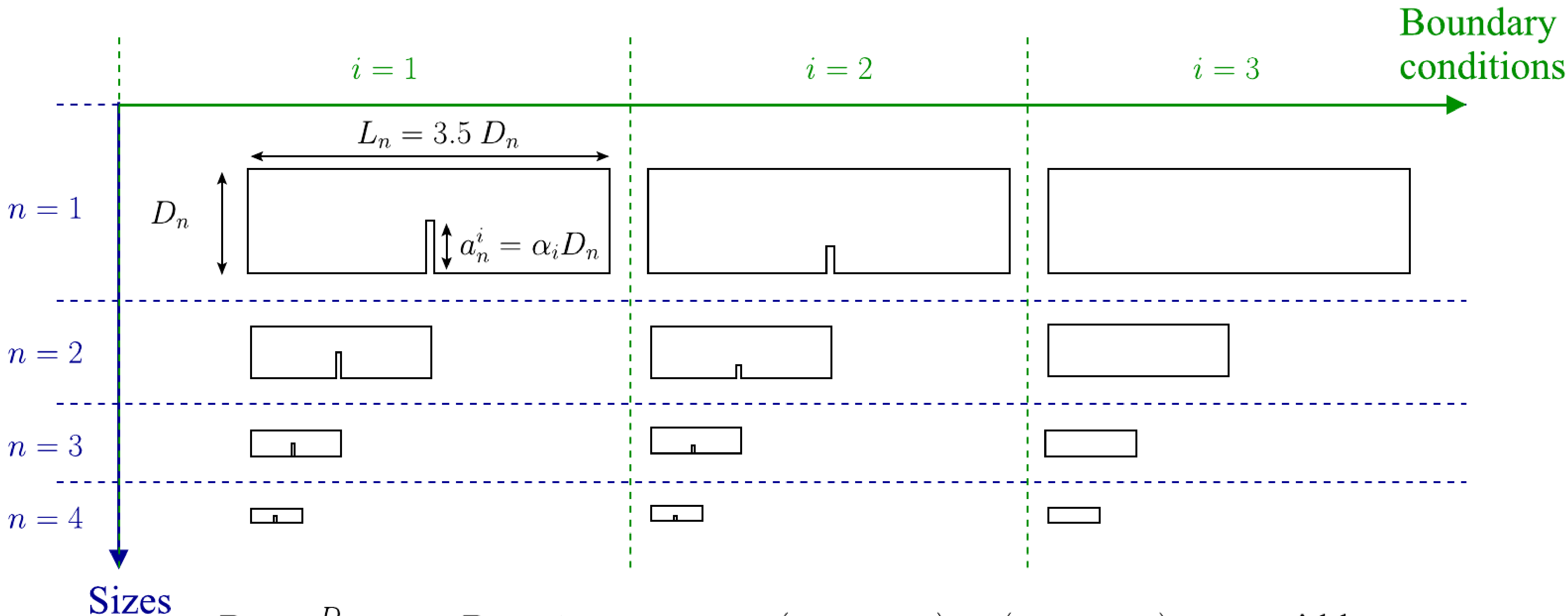
For any given stress, we impose same energy, dissipation and elongation in both models.

Note that the analysis was already carried out with other non-local approach (Cazes et al 2009, Lorentz et al. 2012)

Analysis of size and shape effects in concrete beams

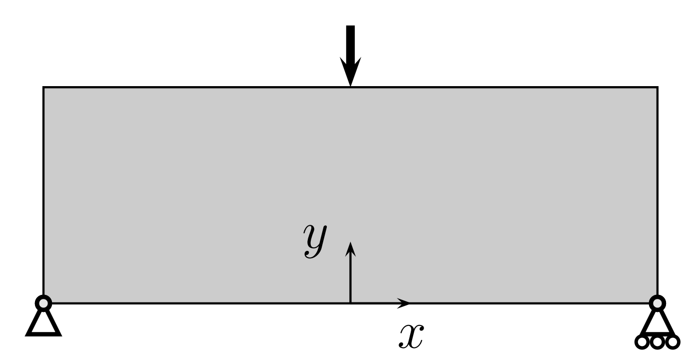
join work with
A. Parrilla-Gomez,
D. Gregoire and G. Pijaudier-Cabot et al. 2017

Size Effect experiments on concrete beams (three point bending)

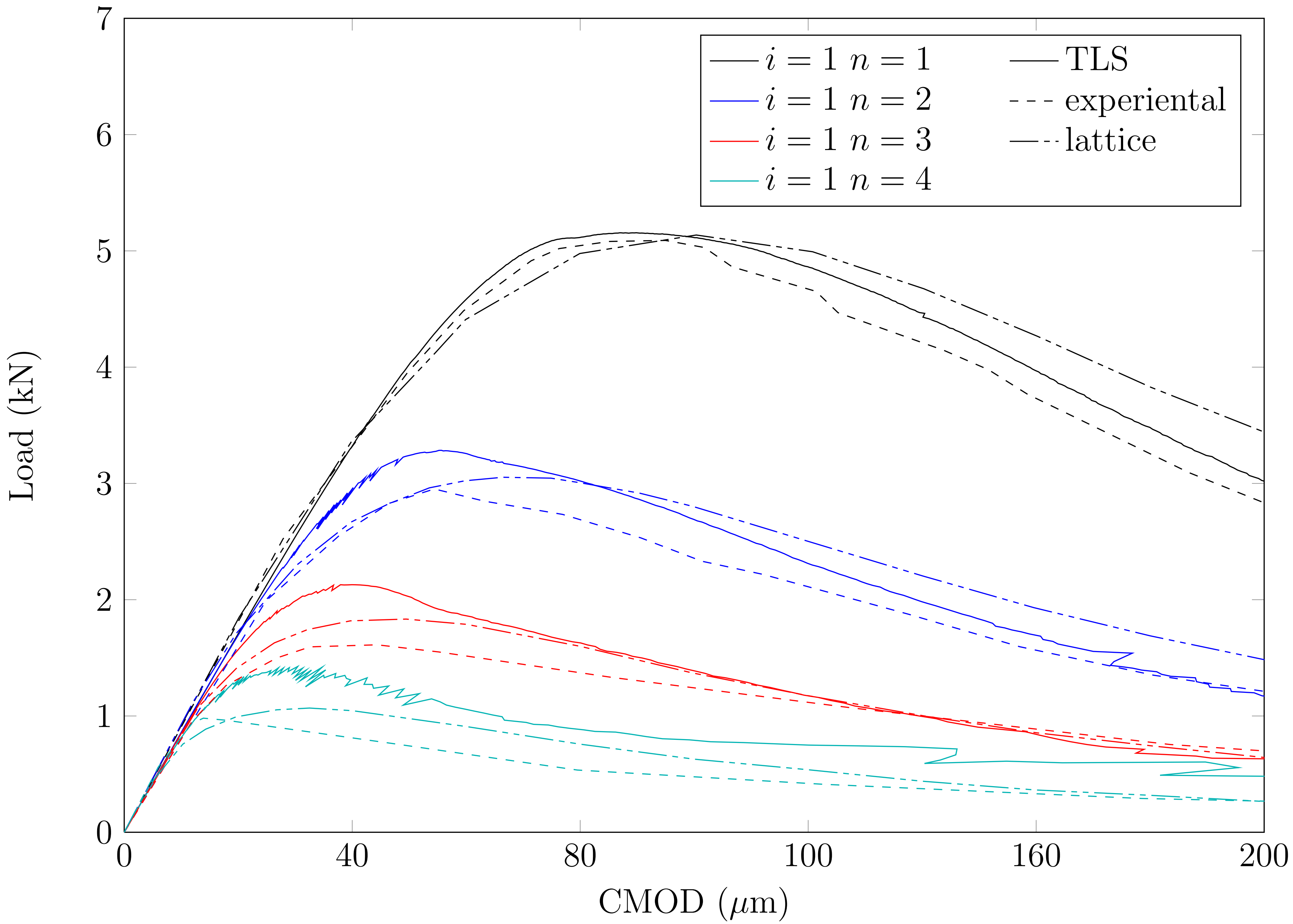


$$D_n = \frac{D_0}{2^{n-1}} \quad ; \quad D_0 = 400\ mm \quad ; \quad (\alpha_1, \alpha_2, \alpha_3) = (0.5, 0.2, 0) \quad ; \quad \text{width} : 50\ mm$$

D. Grégoire, L. Rojas-Solano, and G. Pijaudier-Cabot, "Failure and size effect for notched and unnotched concrete beams," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 37, no. 10, pp. 1434–1452, 2013.



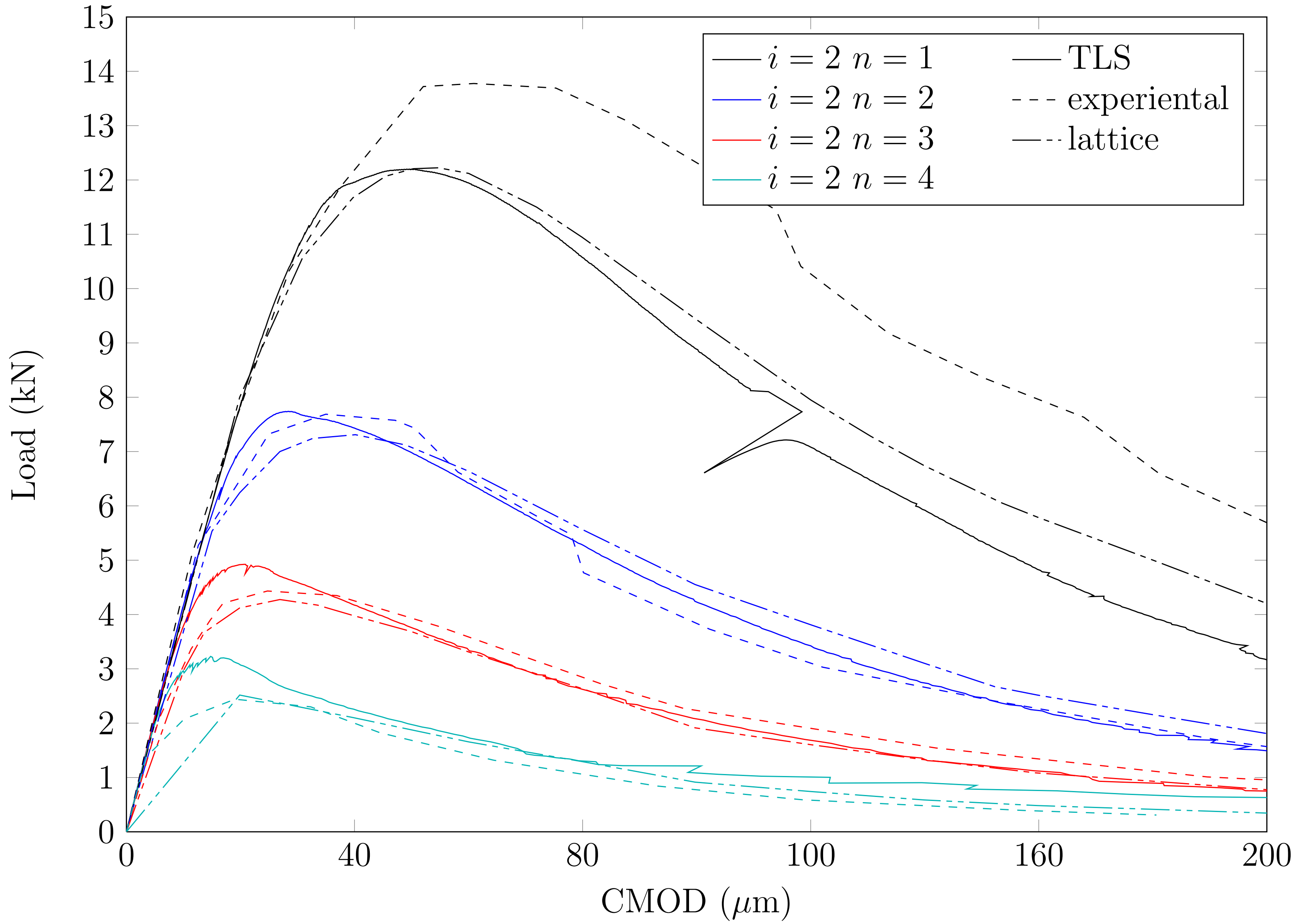
Deep notch



Lattice model:

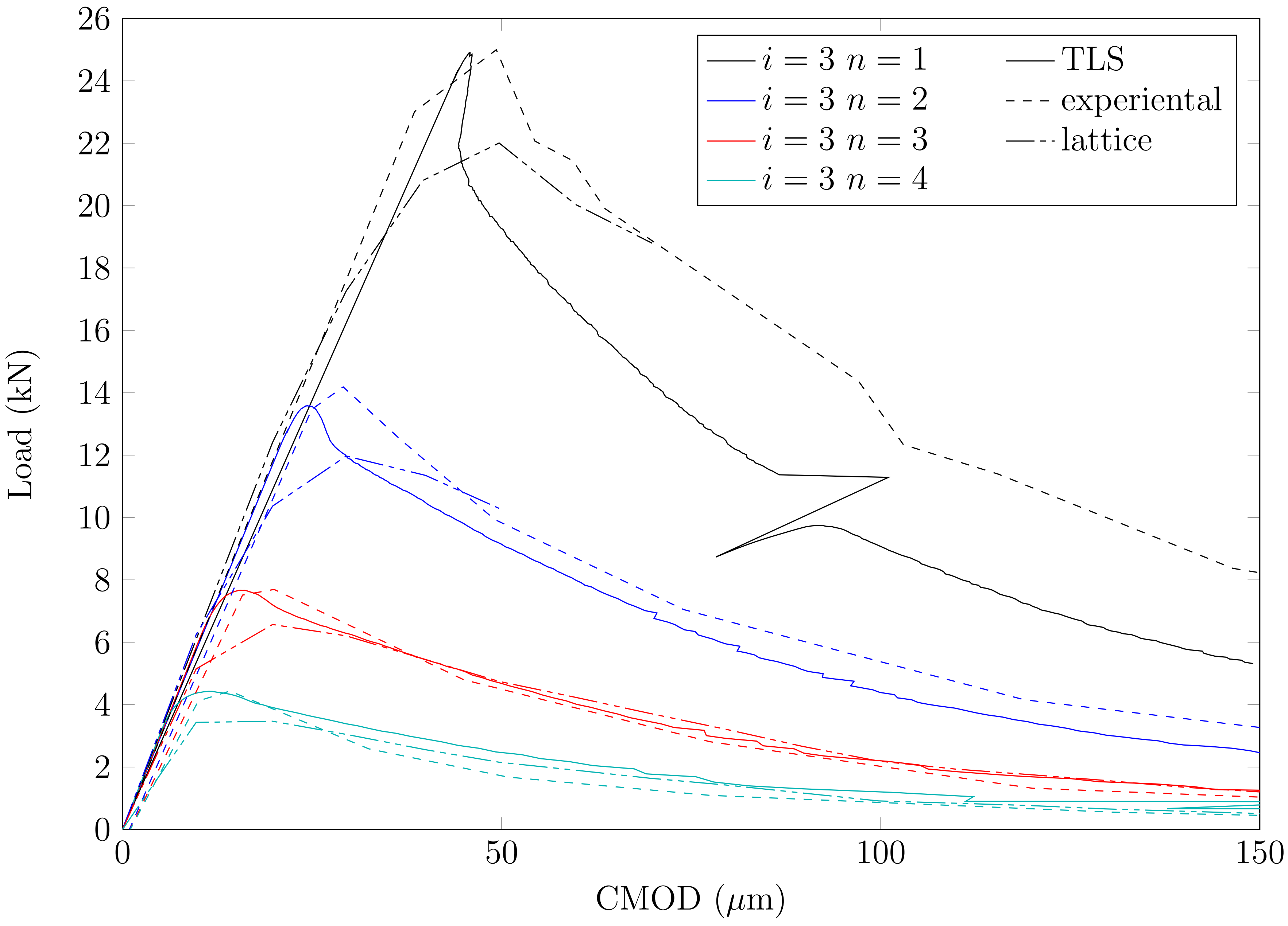
P. Grassl, D. Grégoire, B. Rojas-Solano, Laura, and G. Pijaudier-Cabot, "Meso-scale modelling of the size effect on the fracture process zone of concrete," *International Journal of Solids and Structures*, vol. 49, no. 13, pp. 1818–1827, 2012.

Small Notch



Load (kN)
CMOD (μm)
0
40
80
100
160
200
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
i = 2 n = 1
i = 2 n = 2
i = 2 n = 3
i = 2 n = 4
TLS
experiental
lattice

No notch



26
24
22
20
18
16
14
12
10
8
6
4
2
0
Load (kN)
0
50
100
150
CMOD (μm)
$i = 3\ n = 1$
$i = 3\ n = 2$
$i = 3\ n = 3$
$i = 3\ n = 4$
TLS
experiental
lattice

Summary on TLS (VI)

- The extra non-local numerical work is only in the localizing phase (nothing special in the sane phase).
- Clear indication where to put the crack, giving displacement discontinuity (level set phi = lc)
- Explicit scheme nonlinear solver (robustness of the nonlinear solve).
- “Fast” (2D 5-30 min, 3D 5-10h on 20 procs).

Distinction with gradient damage / phase-field model

- TLS combines sharp crack representation (where crack is fully formed) and diffuse (process zone). There is thus no need for fine mesh along the whole crack path, just in the process zone.
- Eikonal equation instead of a Laplace equation. Three important consequences
  - No matrix solve for damage update, fast marching is used
  - No boundary conditions needed for d
  - The thickness of the localizing band is 2 lc (1D, 2D, 3D)

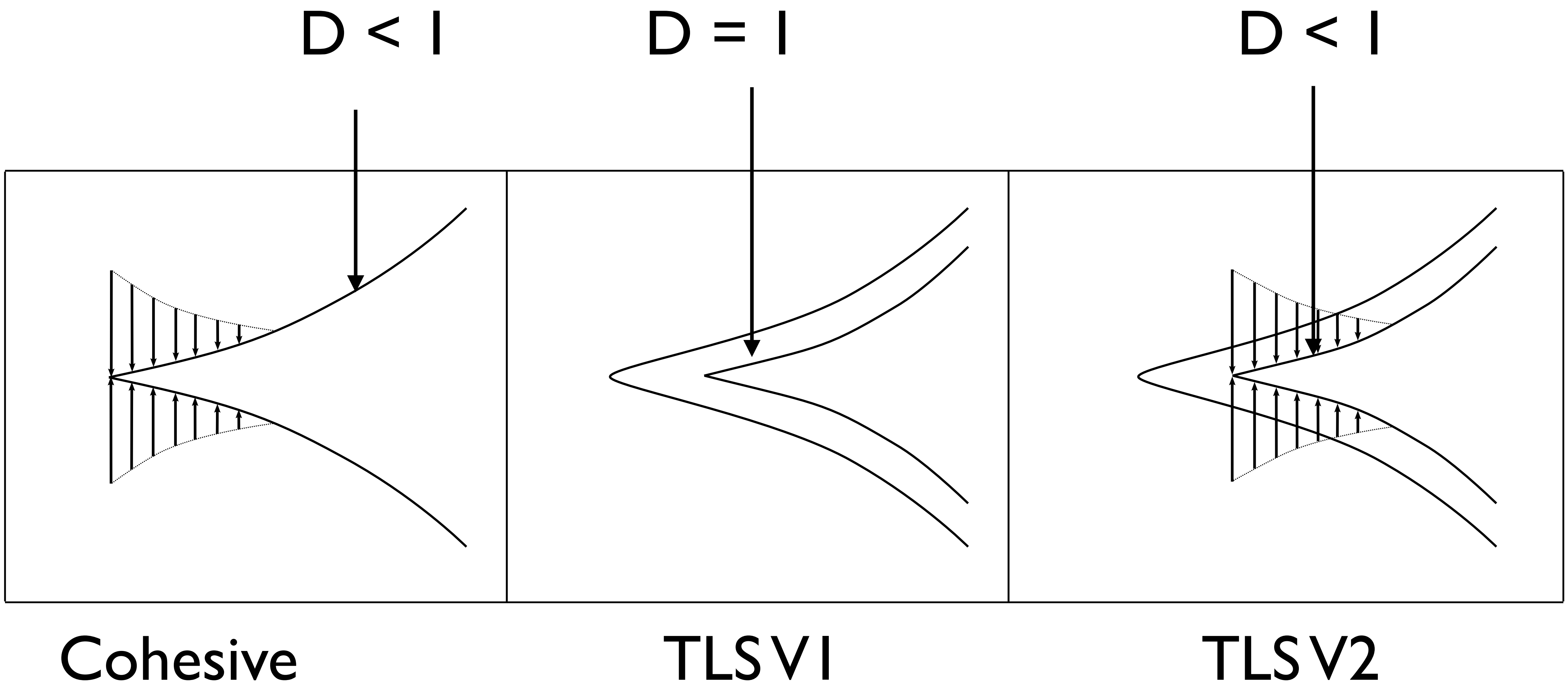
$$\|\nabla D\| = \frac{g(D)}{l_c}$$ TLS

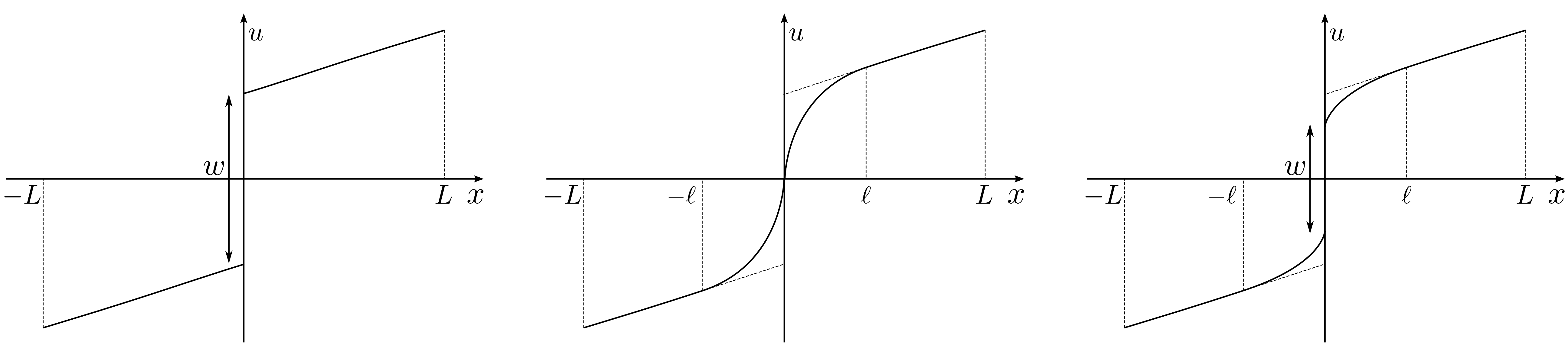
$$\Delta D = \frac{h(D,\epsilon)}{l_c^2}$$ Damage Gradient / PF

Difficulties with TLS V1

- TLS V1 is fine for Griffith type crack and traction free crack
- We noticed that long process required much more element per lc than short ones for the same accuracy.
- In TLS V1, the crack is placed traction free on faces where damage is one. Applying contact with friction afterwards is an issue.
- Failure under compression will be an issue : how to go from a scalar isotropic behavior to a surface oriented localization.

Motivations for a limited softening in the bulk (TLS V2)

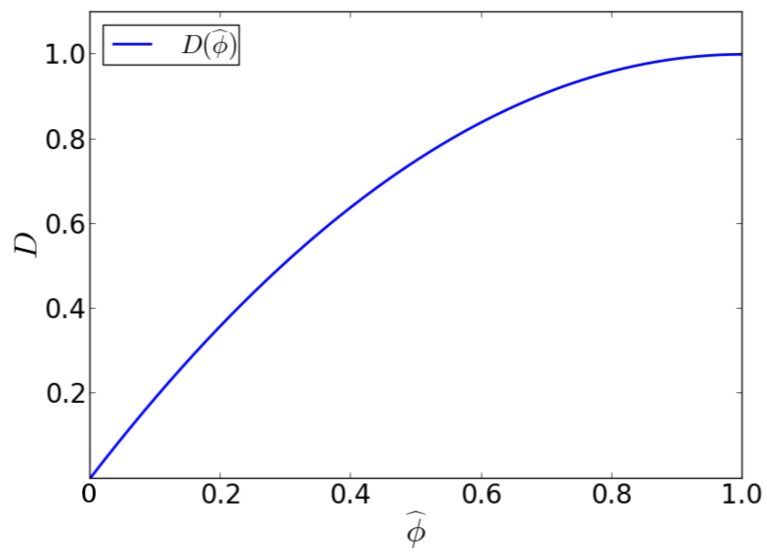




(a) (b) (c)

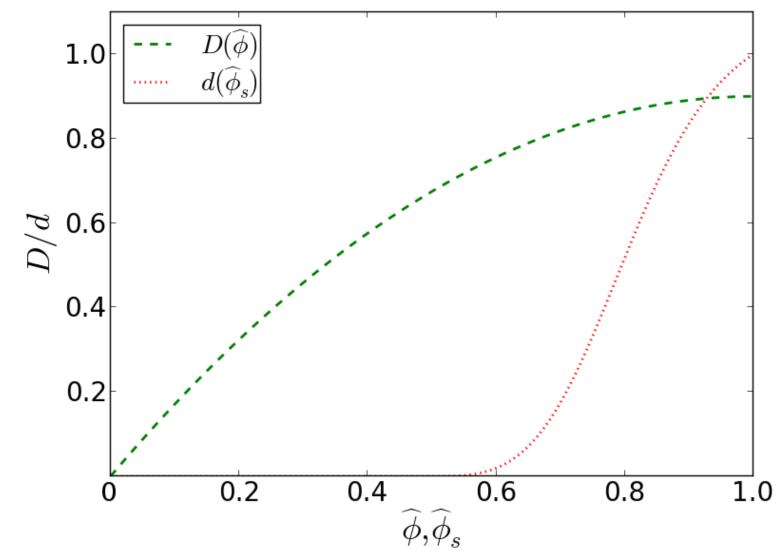
TLS V2 model (cohesive capabilities)

- Bulk damage D is strictly inferior to 1
- Interfacial damage d is also a function of $\phi_s = \phi(x = 0)$, starts to grow for a critical value ϕ^*

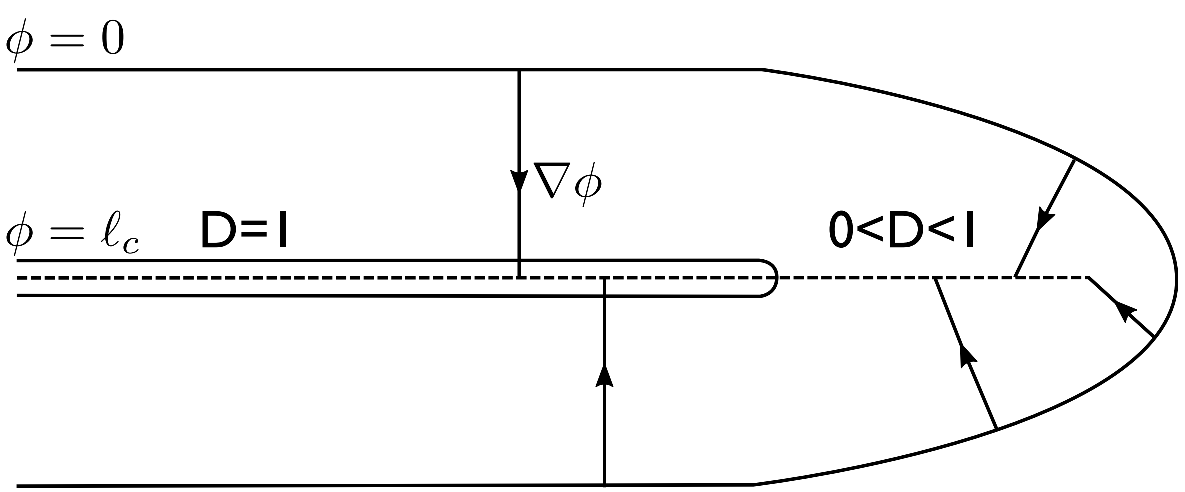


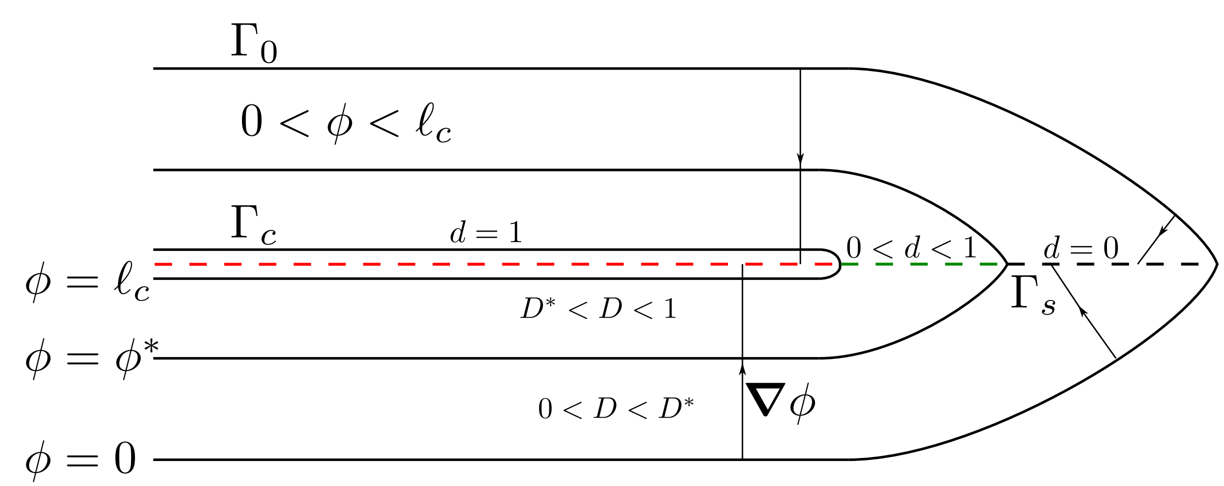
TLS V1

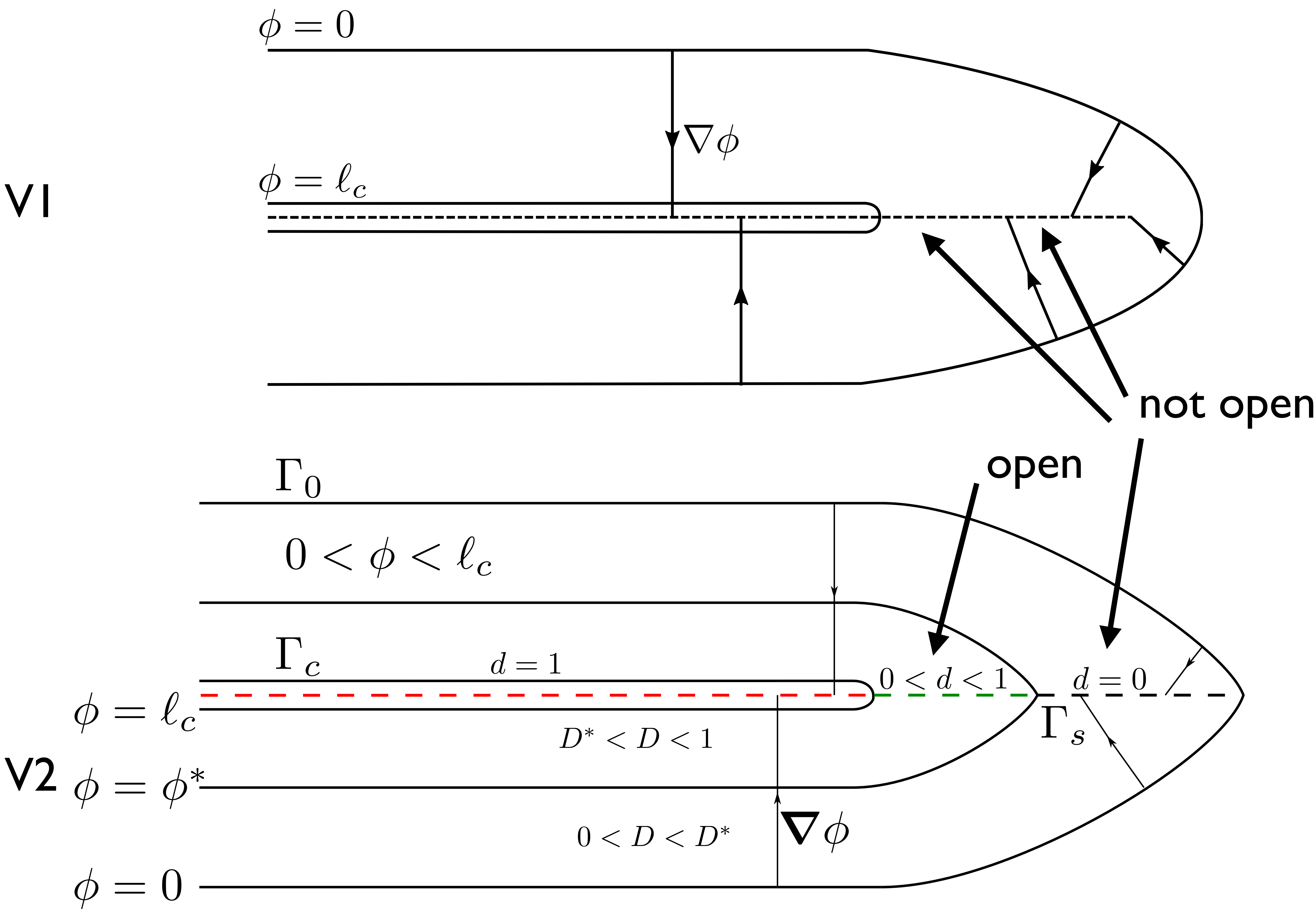
$\Longrightarrow$



TLS V2







V1
$\phi = 0$
$\nabla\phi$
$\phi = \ell_c$
not open
open
V2
Γ_0
$0 < \phi < \ell_c$
Γ_c
$d = 1$
$0 < d < 1$
$d = 0$
$\phi = \ell_c$
Γ_s
$D^* < D < 1$
$\phi = \phi^*$
$\nabla\phi$
$0 < D < D^*$
$\phi = 0$

How to combine interfacila and bulk damage evolution

In TLS V1 bulk damage evolves as

$$\dot{\overline{D}} \geq 0, \quad \overline{Y} - Y_c \overline{H} \leq 0 \quad (\overline{Y} - Y_c \overline{H})\dot{\overline{D}} = 0 \quad \text{and} \quad D = D(\phi)$$

In CZM model interfacial damage evolves as

$$\dot{d} \geq 0, \quad y - y_c h(d) \leq 0, \quad (y - y_c h(d))\dot{d} = 0$$

TLS V2 states that $d = d(\phi\,|_{\text{interface}})$

So interfacial and bulk damage cannot evolve independently, they are tied by the level set

The TLS V2 evolution is based on configurational forces

Level set field evolution condition in the TLS V2

As the level set evolves it dissipates both in the bulk and the interface.
We impose that this loss is equal to the critical value for level set advance

- Configurational force:

$$g = \int_0^{\phi_s} Y D'(\phi)\,\mathrm{d}x + \frac{1}{2} y d' \Big|_{\phi=\phi_s}$$

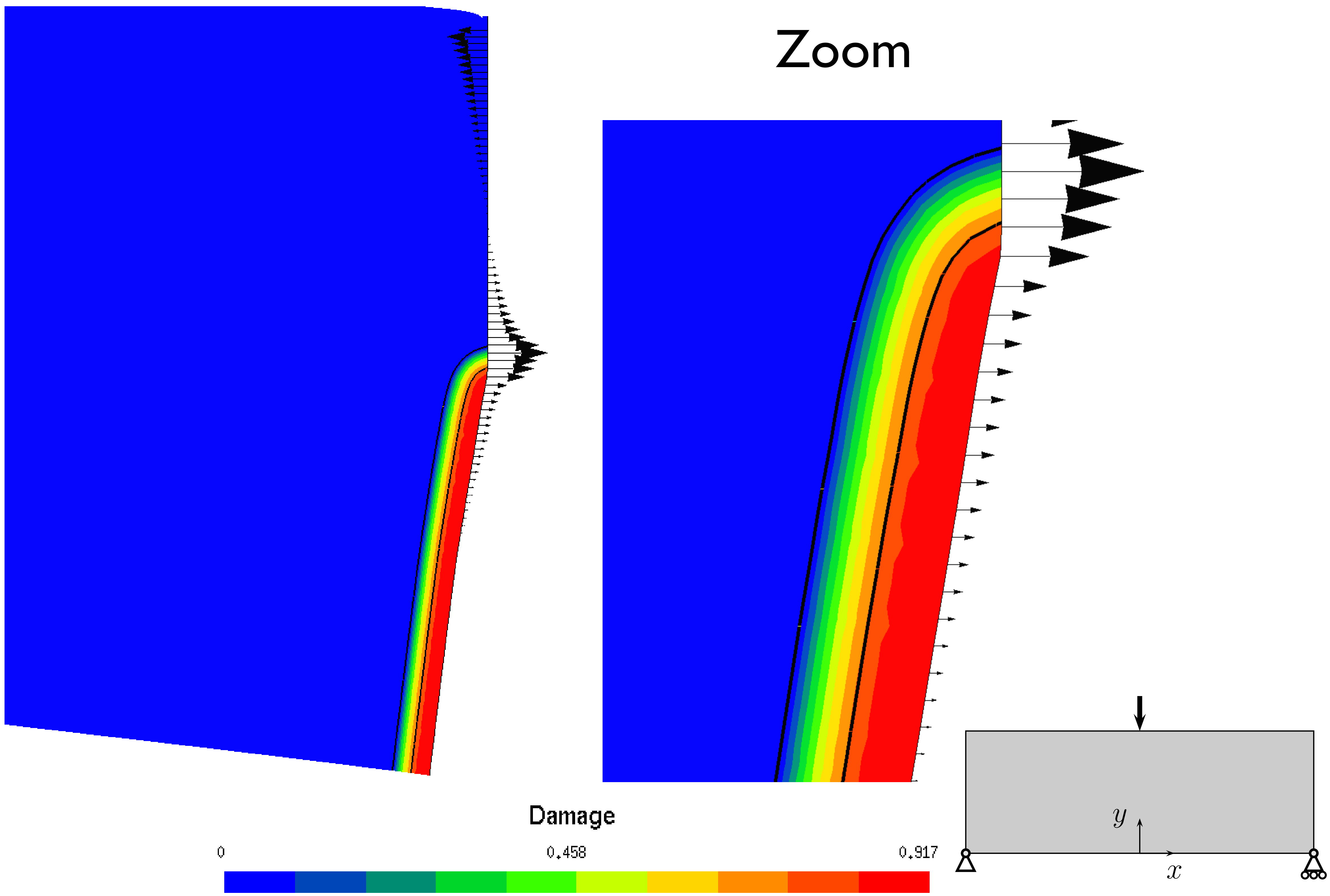
- Critical value:

$$g_\mathrm{c} = \int_0^{\phi_s} Y_\mathrm{c} H(D(\phi)) D'(\phi)\,\mathrm{d}x + \frac{1}{2} y_\mathrm{c} h(d) d' \Big|_{\phi=\phi_s}$$

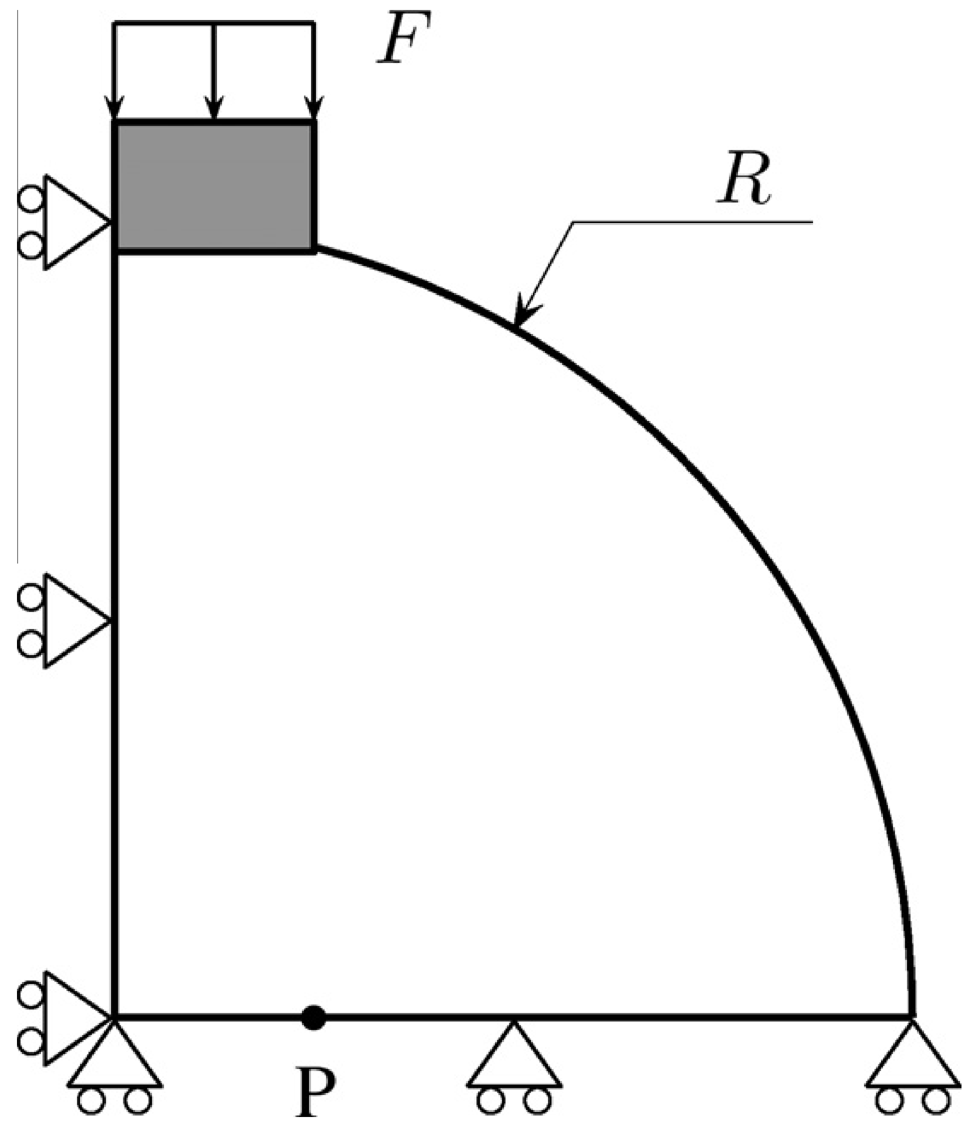
- Evolution laws:

$$\dot{\phi} \geq 0, \quad g - g_\mathrm{c} \leq 0, \quad (g - g_\mathrm{c})\dot{\phi} = 0$$

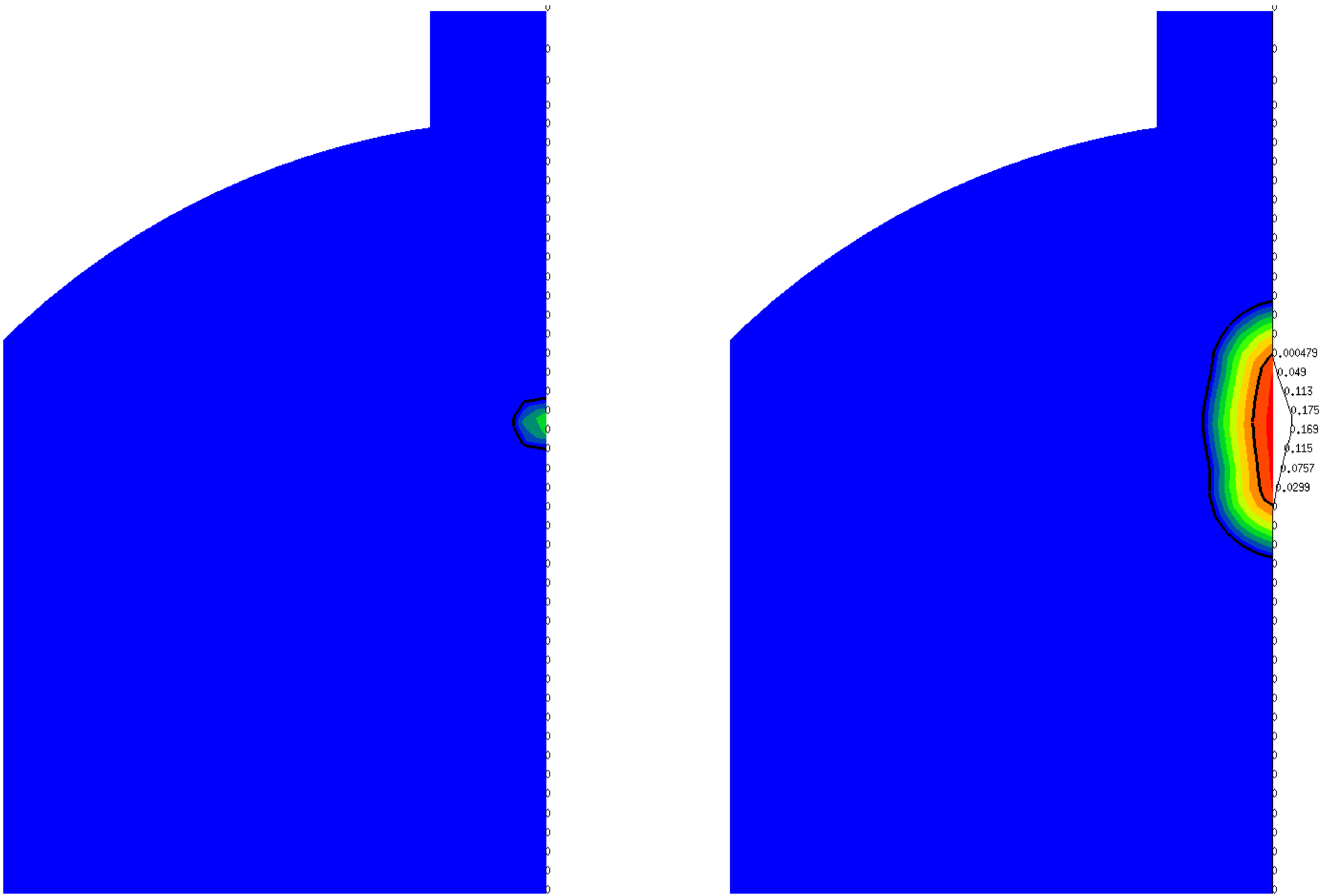
TLS V2: Damage field and cohesive forces



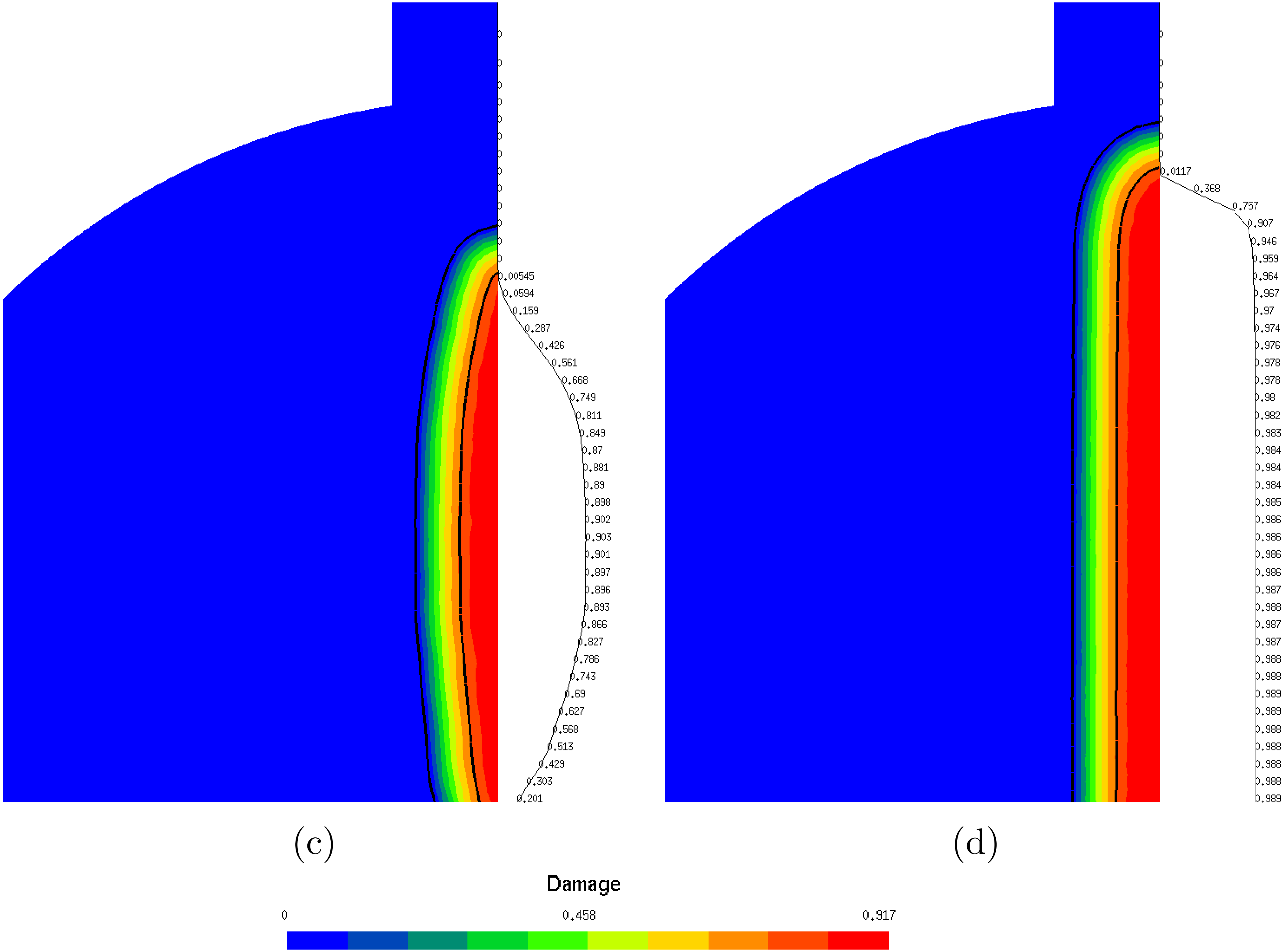
Splitting test (Brazilian test)



Bulk and cohesive damage



(a) (b)



(c) (d)

Direct access to crack opening

Conclusions

- TLS lies between damage and cohesive zone models (best of both worlds). It gives CZM a way to propagate on its own branch and coalesce.
- Crack appears automatically (location is part of the TLS model).
- The TLS theory is implemented using the X-FEM to allow for displacement jumps in the simulation (remeshing should be possible).
- No matrix solve for damage update and localization treatment very limited in space -> low CPU.

Other Works

- Fracture Dynamics (no matrix solve at all and fixed grid).
- Ductile failure (ongoing). The cumulative plasticity is controlled.
- Two-scale solver to further reduce computing time. Target: 2D < 5min 1 proc, 3D < 1h 20 proc.